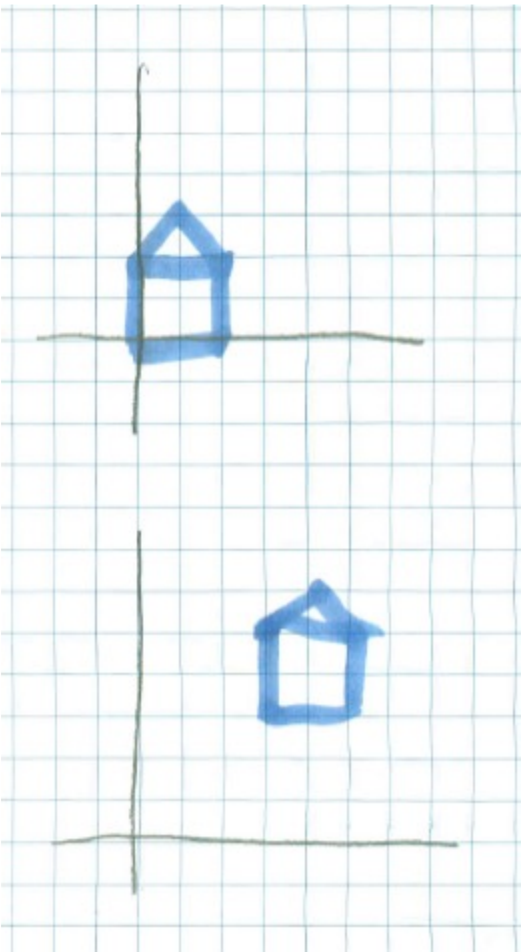
CSE160 – Transformations

- How can I move objects?
- Transforms in 2D
- Matrix: Transforms in 2D
- 2D homogeneous coordinates
- Matrix order matters
- 3D transforms
- Hierarchical transforms
- Assignment 1
- Administrative
- Q&A

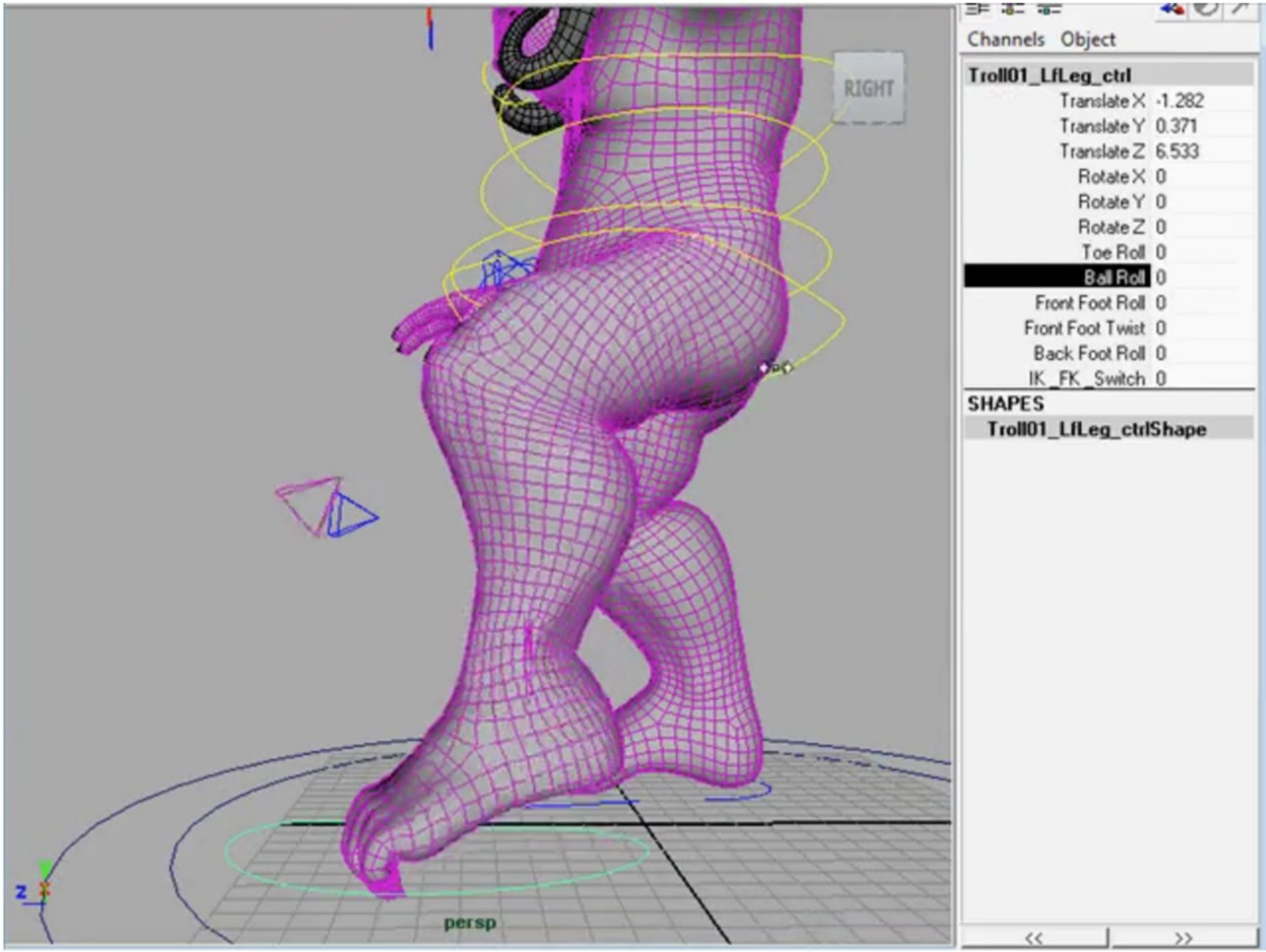
How can I move objects?

Transformations

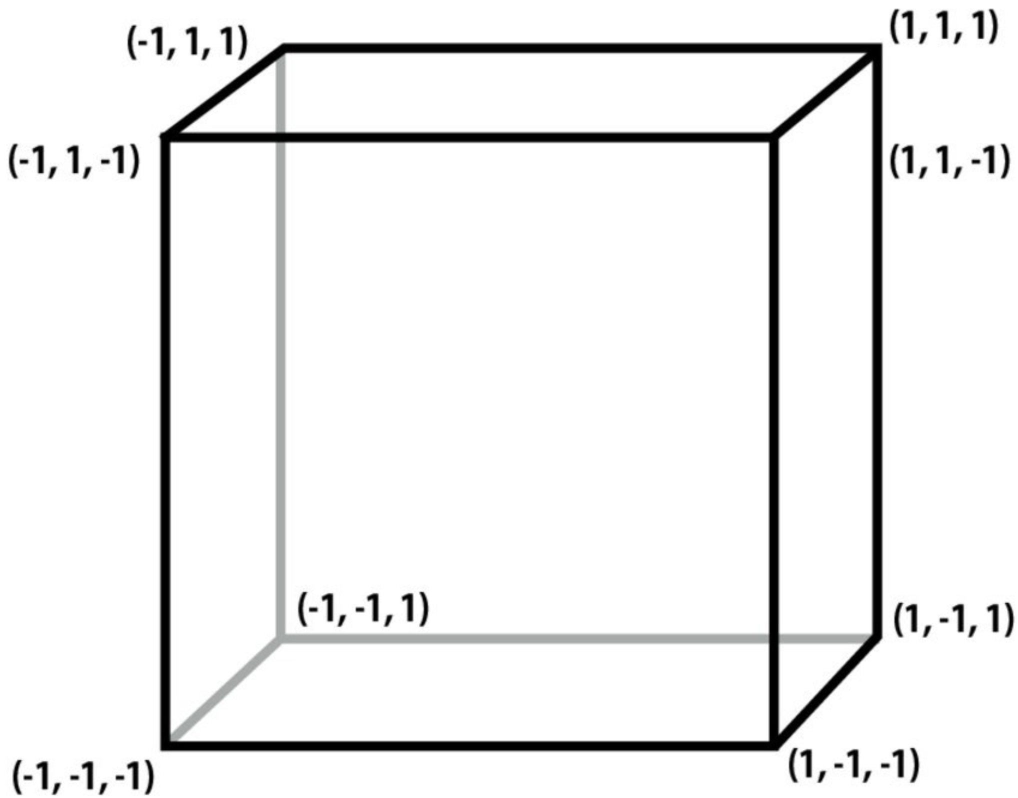


```
void drawHouse() {
 glBegin(GL_QUADS);
       glVertex2d(0,0);
       glVertex2d(0,1);
       glVertex2d(1,1);
       glVertex2d(1,0);
glEnd();
// …. Lots more stuff
}

void main() {
// Draw house at origin
drawHouse();

// Draw house somewhere else
????
}
```

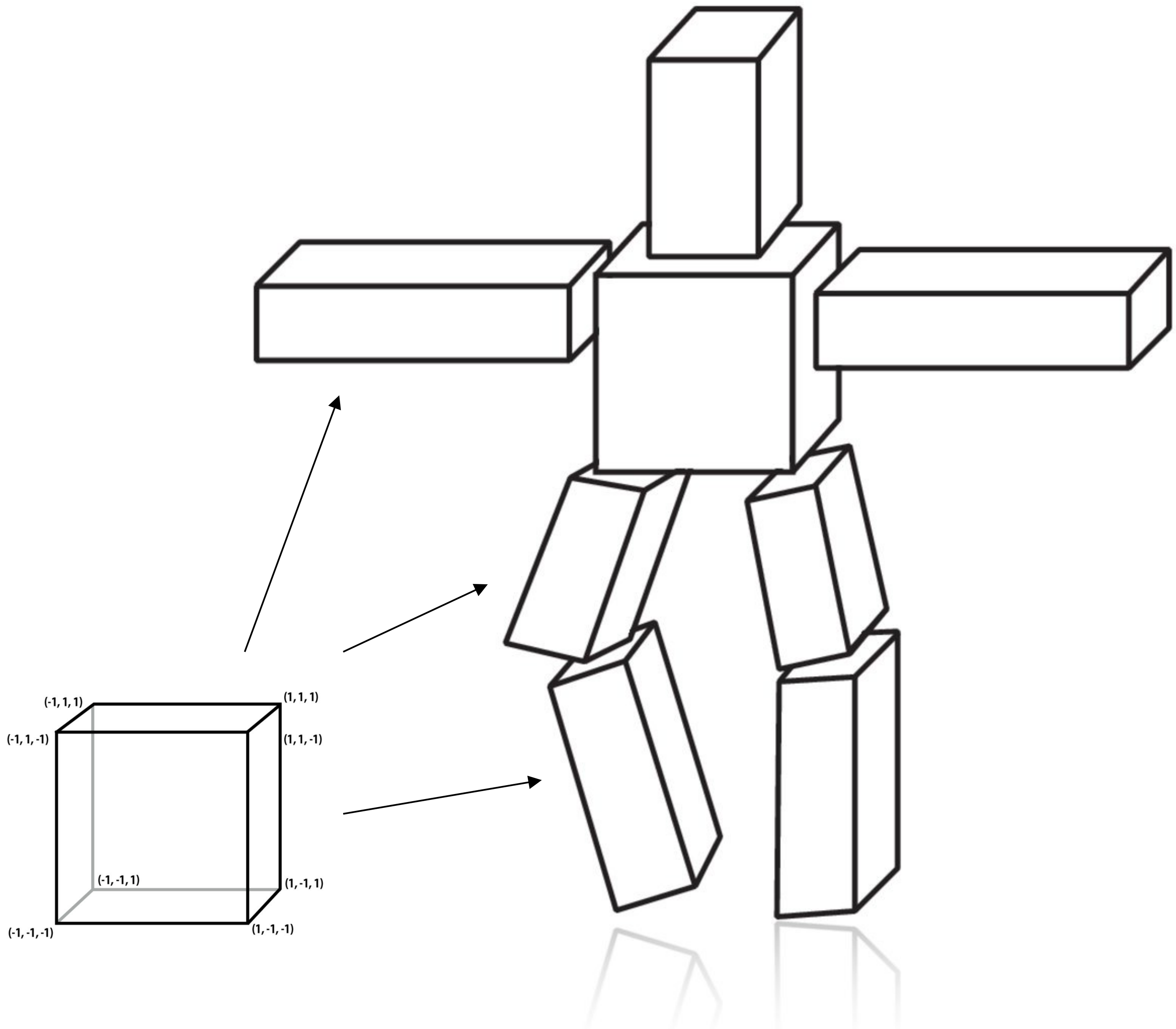
Transformations in character rigging



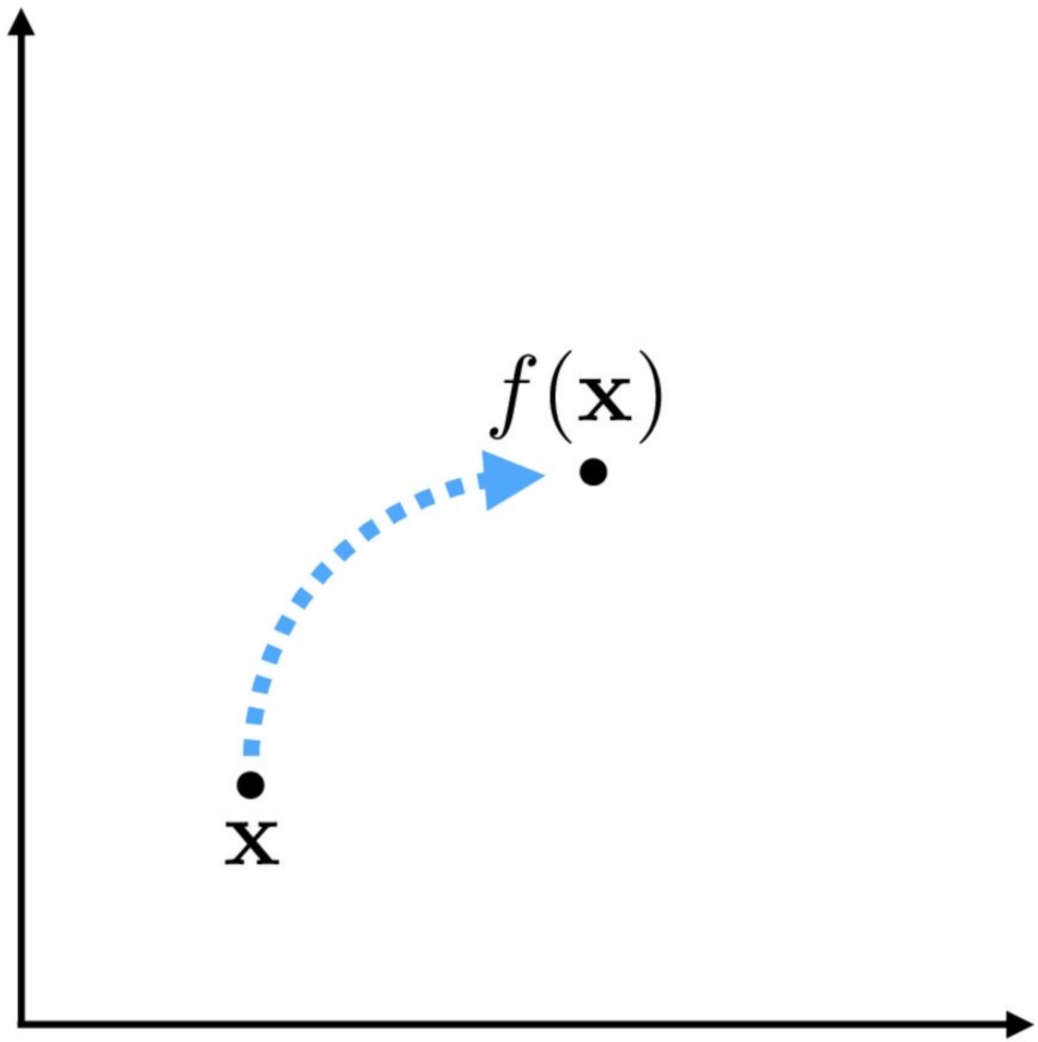
Cube



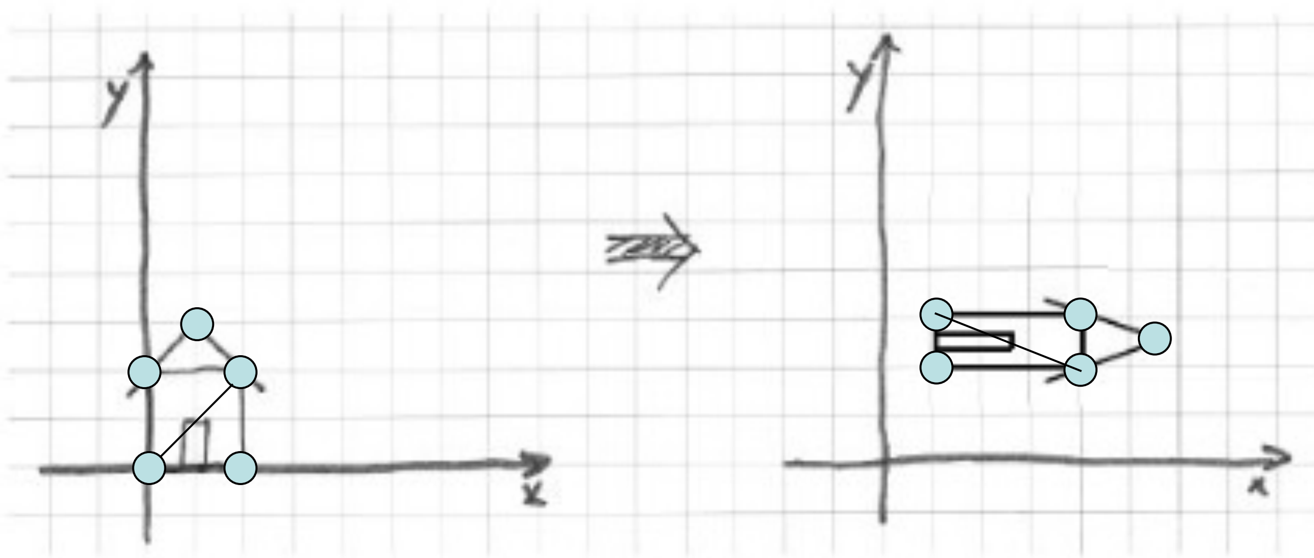
Consider drawing a cube person



Basic idea: f transforms $\mathbf{x}$ to $f(\mathbf{x})$



Exercise: Find the function()



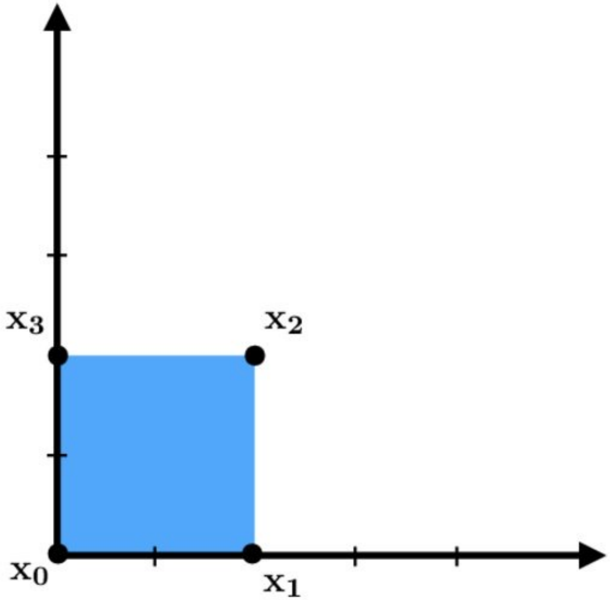
Vertex buffer array
0,0
2,2
2,0
0,2
1,3
2,2
….

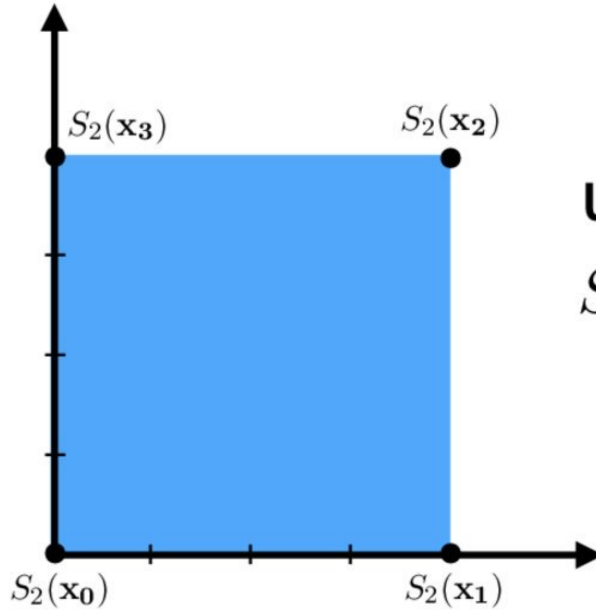
→ f() →

Vertex buffer array
1,3
4,2
1,2
4,3
5.5, 2.5
4,2
….

Transforms in 2D

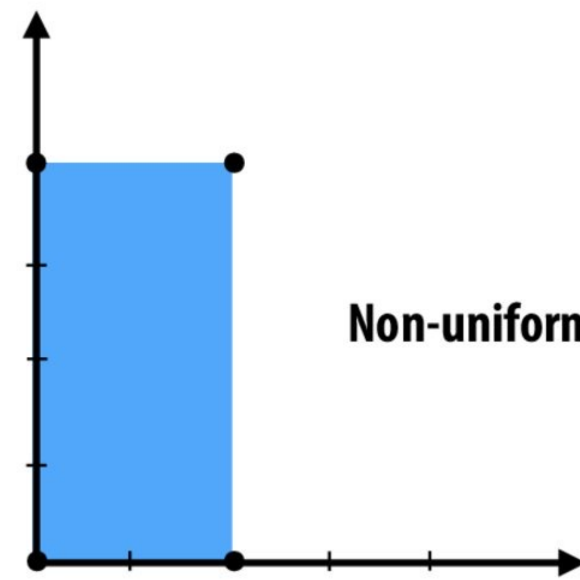
Scale





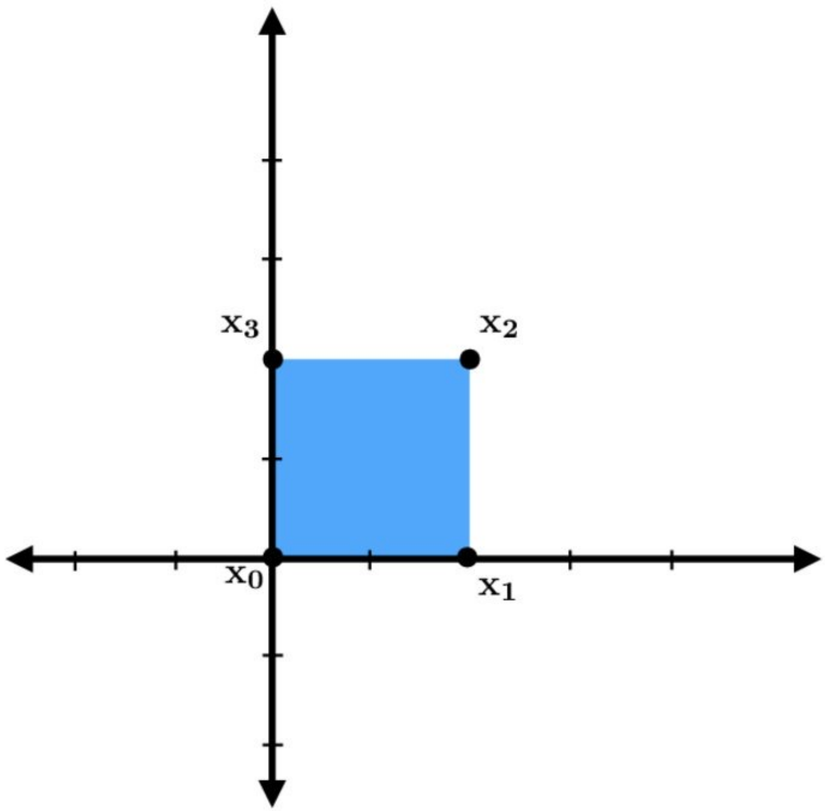
Uniform scale:

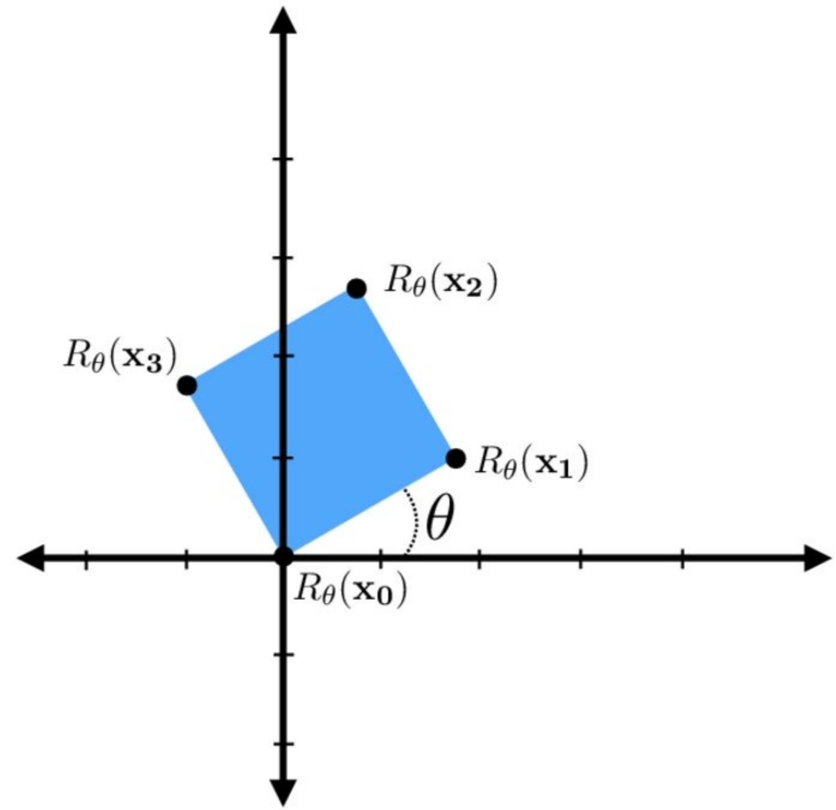
$$S_a(\mathbf{x}) = a\mathbf{x}$$



Non-uniform scale??

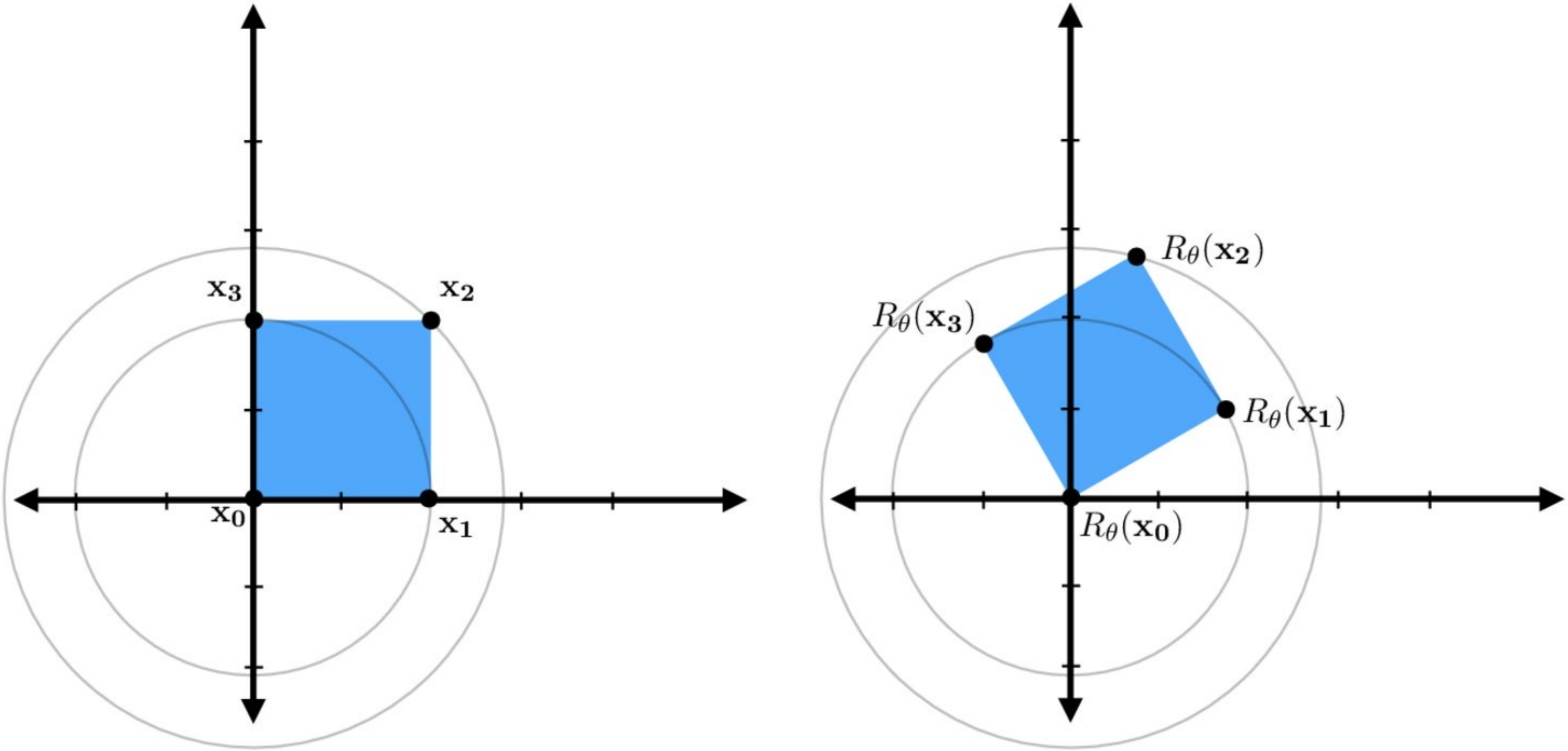
Rotation





R_θ = **rotate counter-clockwise by** θ

Rotation as circular motion

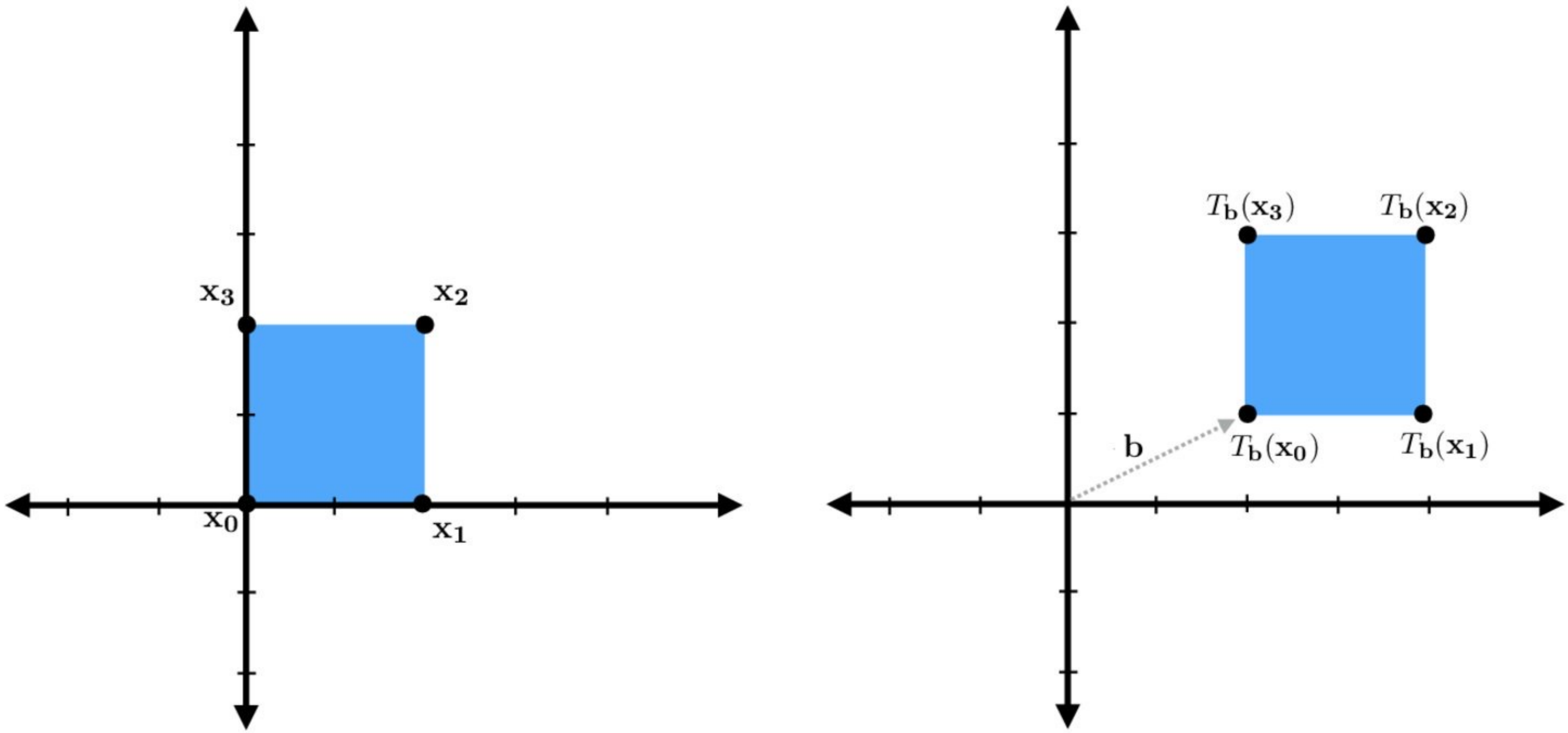


R_θ = **rotate counter-clockwise by** θ

As angle changes, points move along *circular* trajectories.

Hence, rotations preserve length of vectors: $|R_\theta(\mathbf{x})| = |\mathbf{x}|$

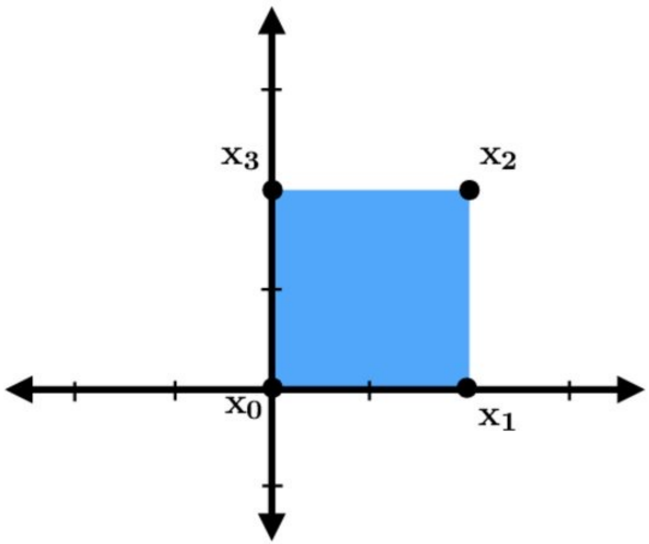
Translation

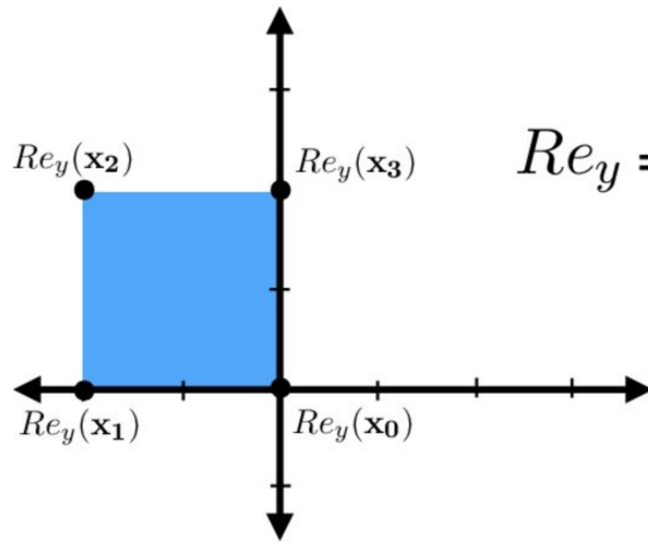


$T_{\mathbf{b}}$ — **"translate by b"**

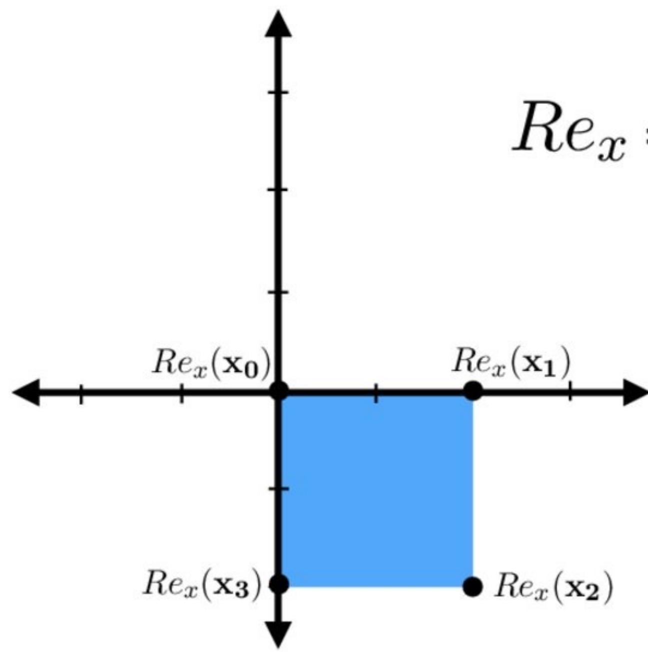
$T_{\mathbf{b}}(\mathbf{x}) = \mathbf{x} + \mathbf{b}$

Reflection



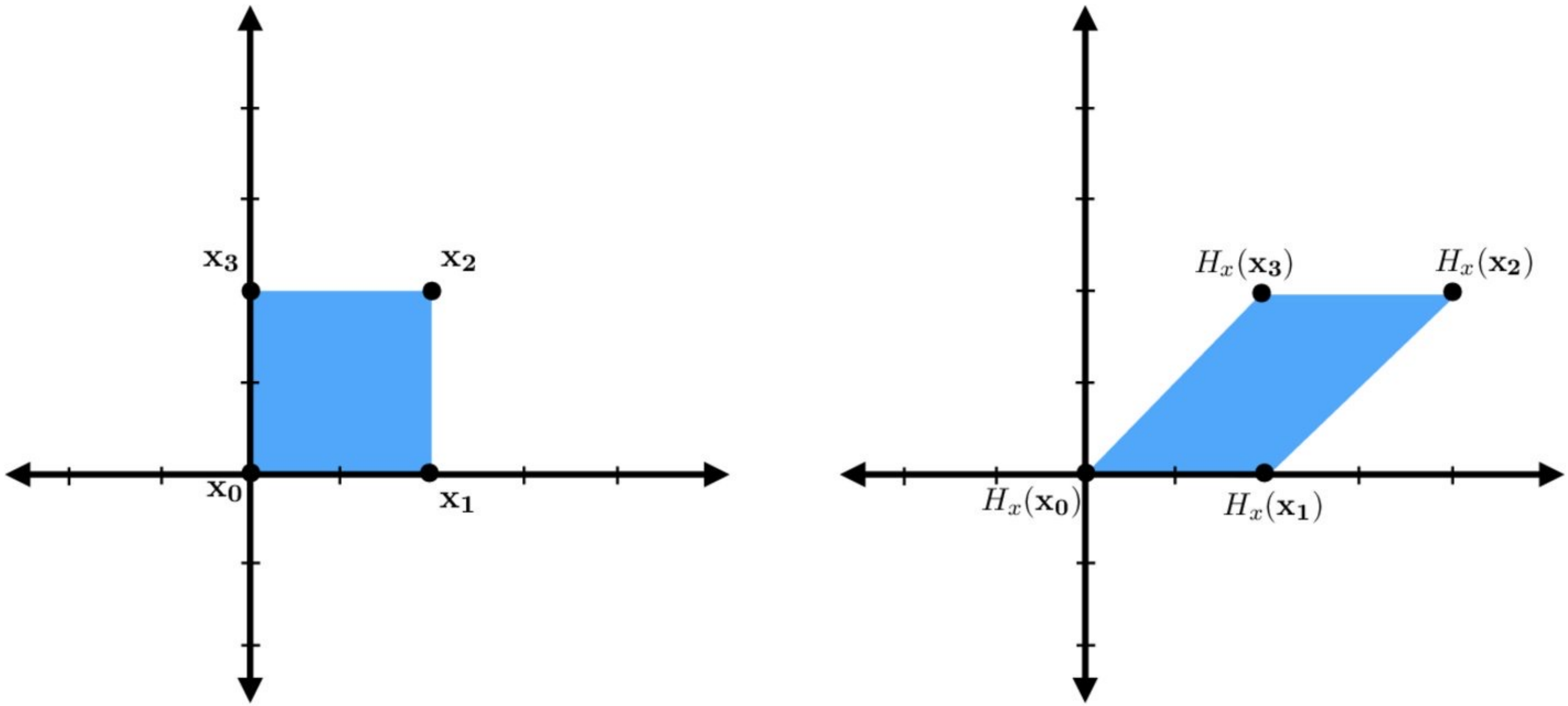


Re_y = **reflection about** y

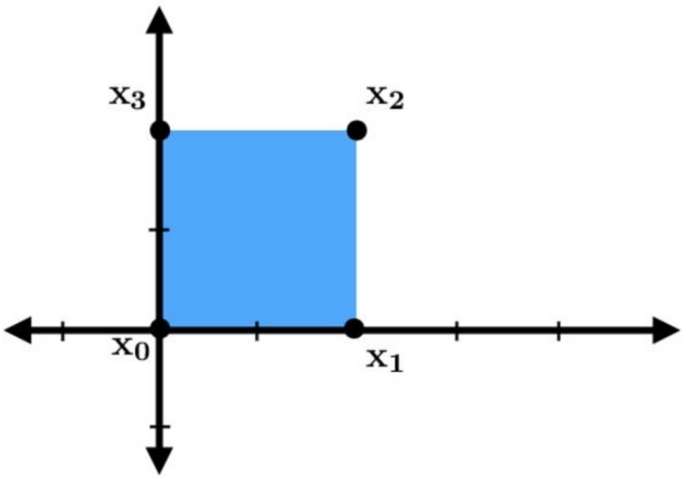


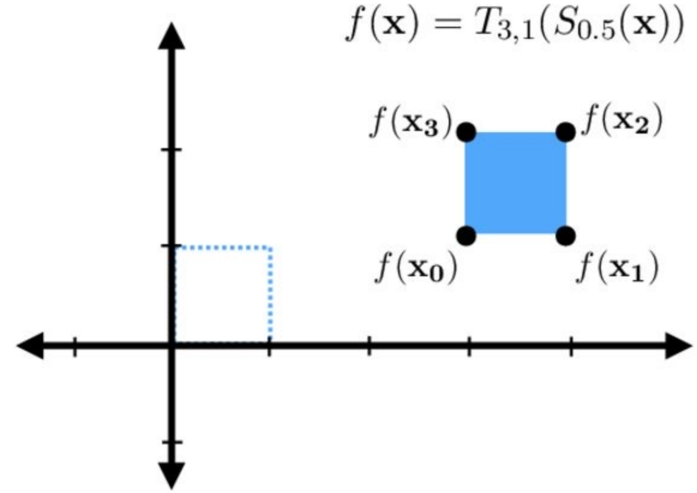
Re_x = **reflection about** x

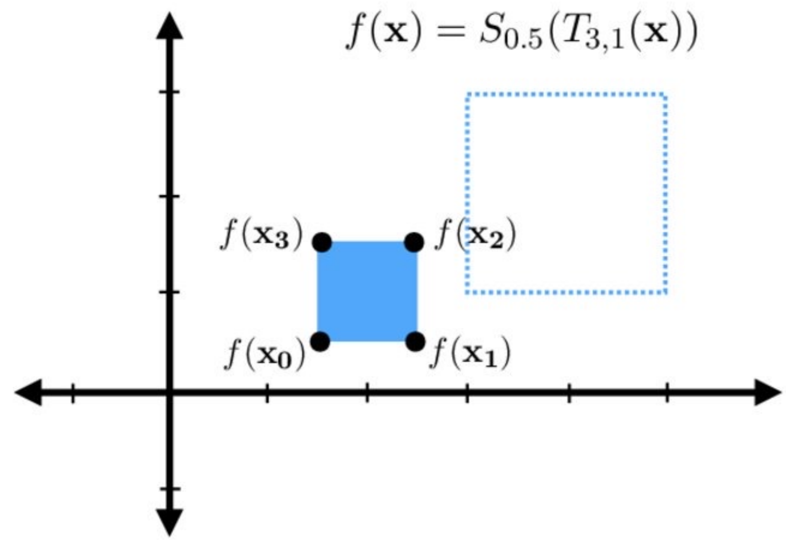
Shear (in x direction)



Compose basic transformations to construct more complicated ones



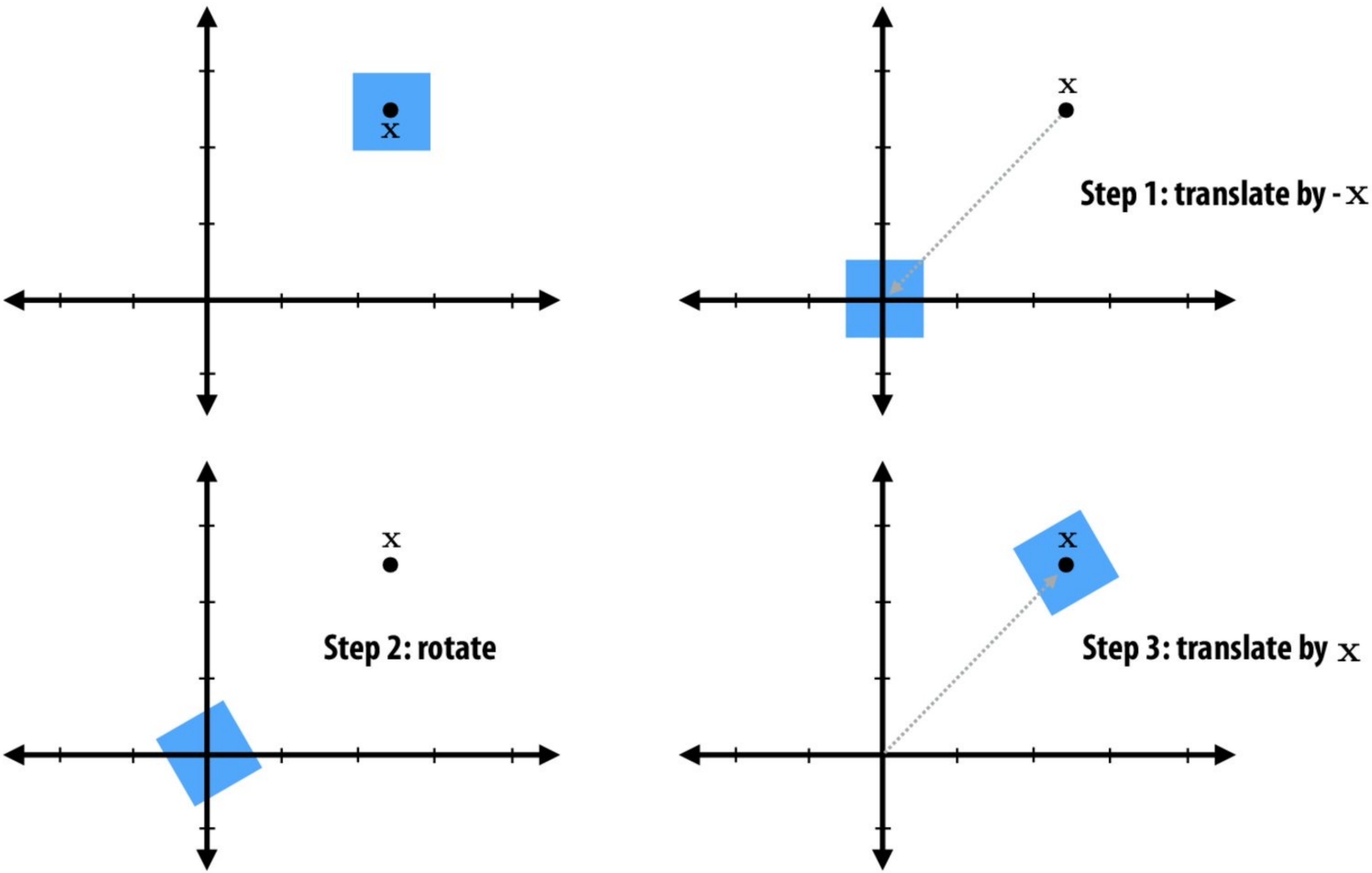




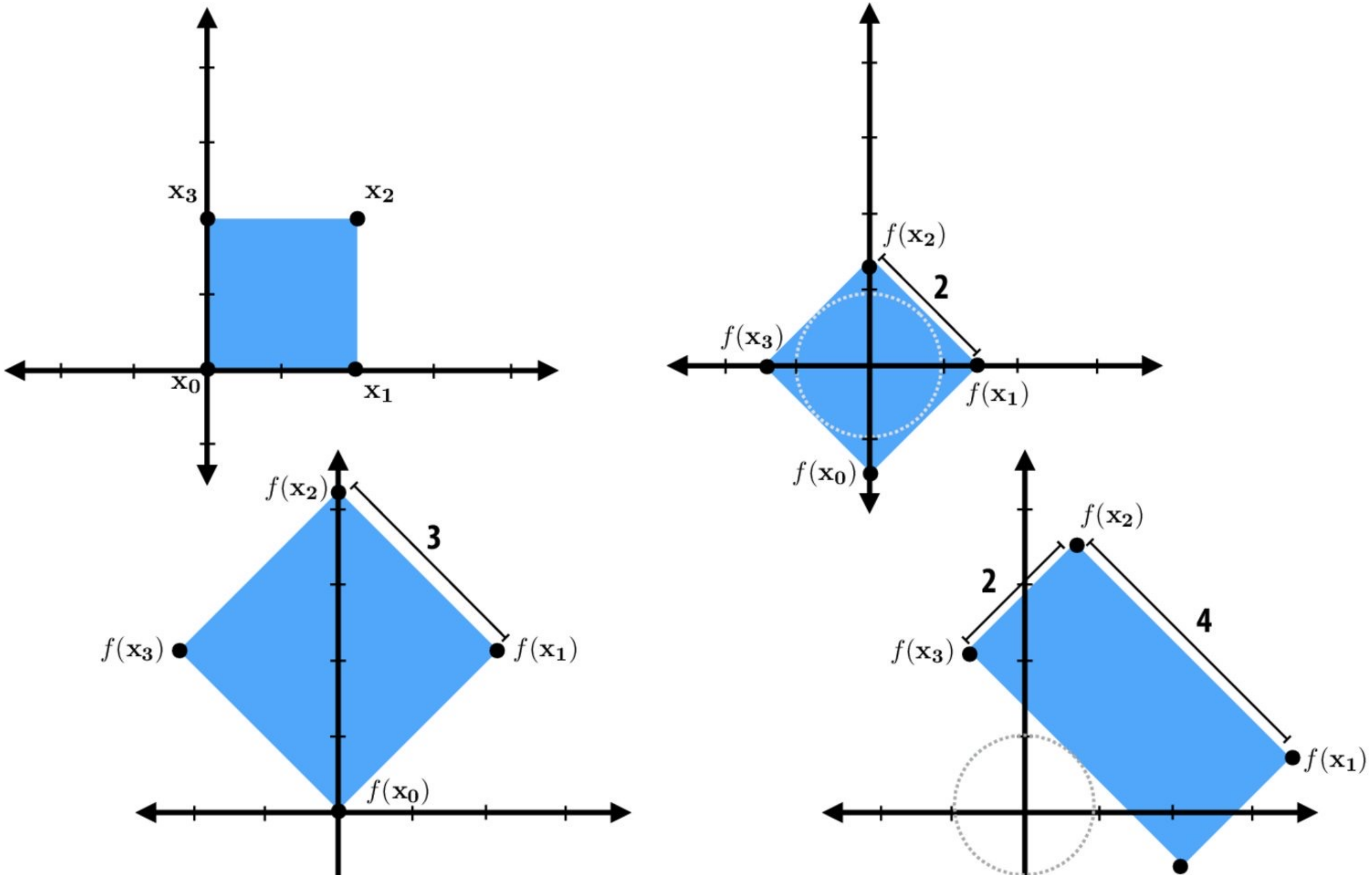
Note: order of composition matters

Top-right: scale, then translate
Bottom-right: translate, then scale

Common task: rotate about a point x



How would you perform these transformations?



Usually more than one way to do it!

Matrix: Transforms in 2D

Scaling in 2D

Scaling

- Multiply each coordinate by a constant amount:

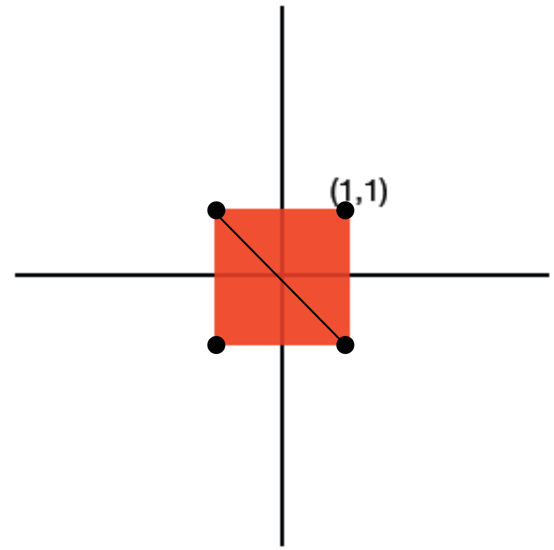
$$x' = rx$$
$$y' = sy$$

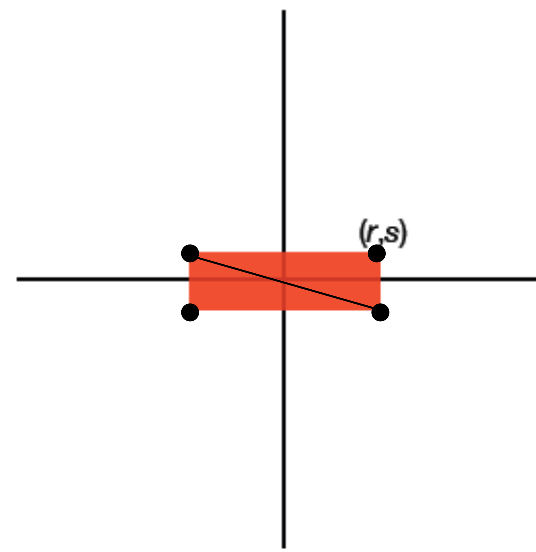
- Use matrix notation for compactness:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{p}' = \mathbf{S}\mathbf{p}$$

- Scaling is a linear transformation





Translation in 2D

Translation

- Offset each coordinate by constant amount:

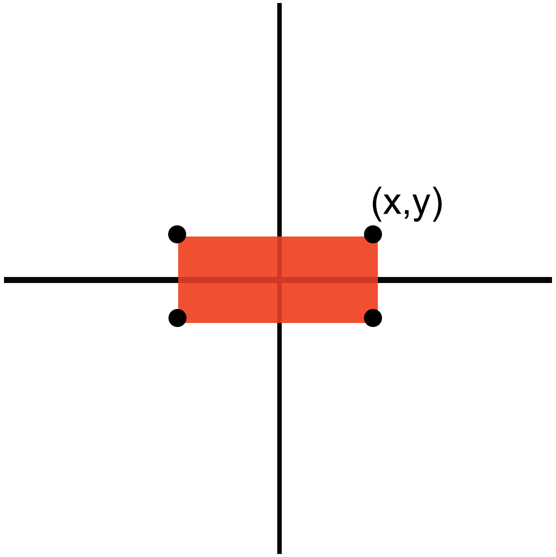
$$x' = x + \Delta x$$
$$y' = y + \Delta y$$

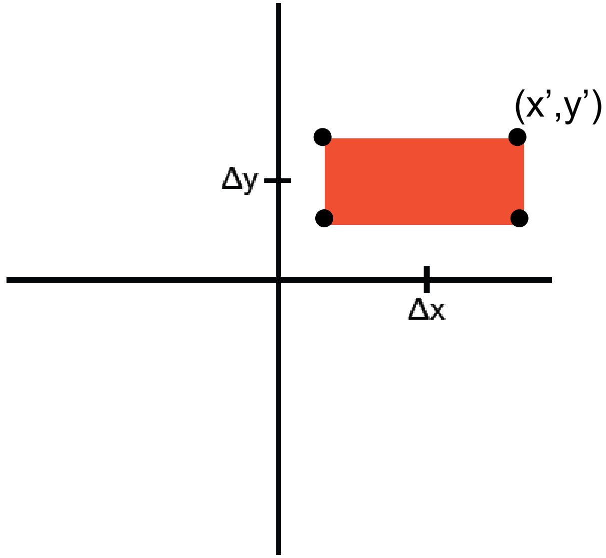
- Use vector notation for compactness:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\mathbf{p}' = \mathbf{p} + \mathbf{d}$$

- Translation is an affine transformation, but not a linear transformation





Rotation in 2D

Rotation

- Rotate about the origin by some angle:

$$x = \rho \cos \phi \qquad x' = \rho \cos(\phi + \theta)$$
$$y = \rho \sin \phi \qquad y' = \rho \sin(\phi + \theta)$$

- Use polar coordinates to solve for new positions:

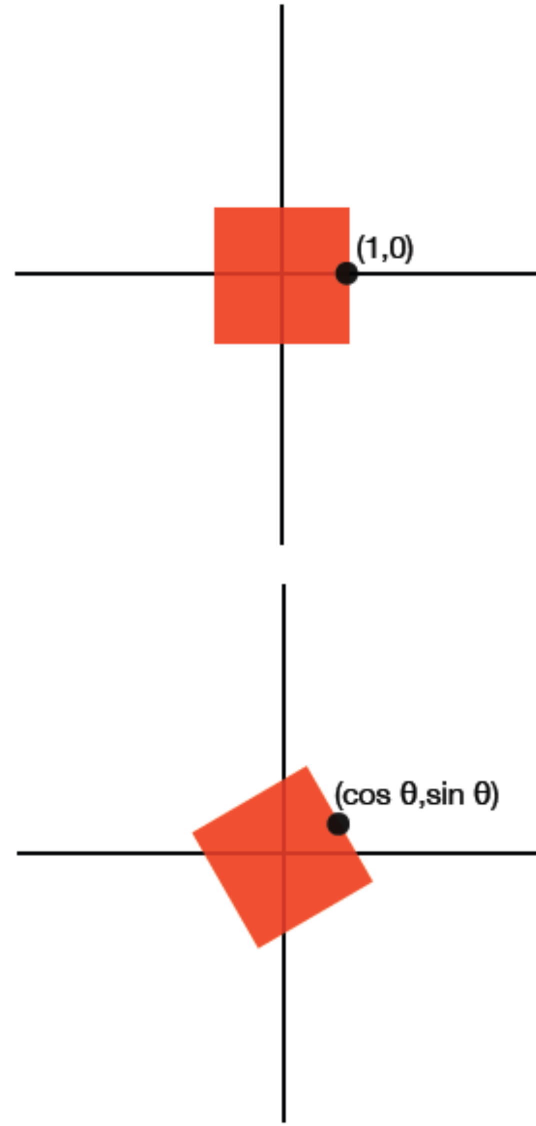
$$x' = x \cos \theta - y \sin \theta$$
$$y' = x \sin \theta + y \cos \theta$$

- Use matrix notation for compactness:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ sin\theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{p}' = \mathbf{R}\mathbf{p}$$

- Rotation is a linear transformation

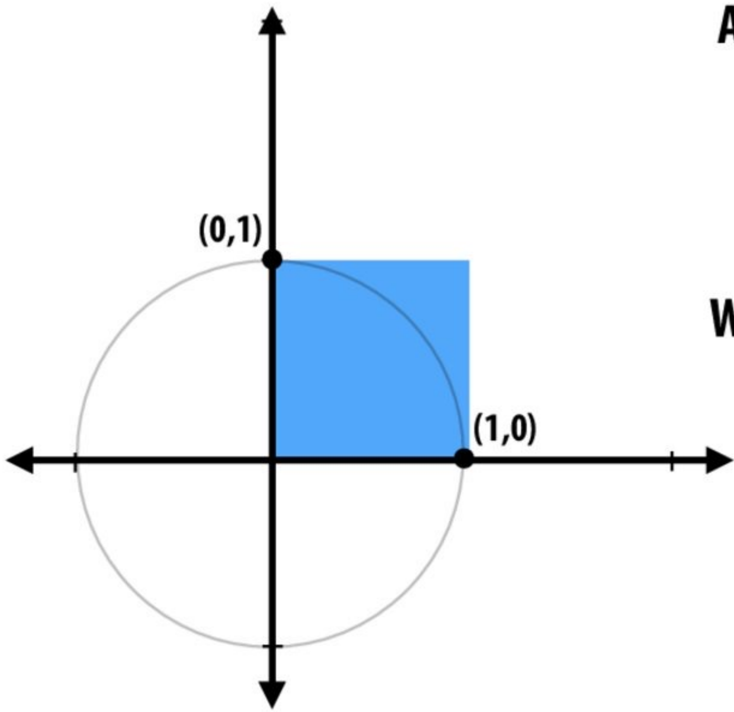


Rotation matrix (2D)

Question: what happens to (1, 0) and (0,1) after rotation by θ?

Reminder: rotation moves points along circular trajectories.

(Recall that $\cos\theta$ and $\sin\theta$ are the coordinates of a point on the unit circle.)



Answer:

$$R_\theta(1,0) = (\cos(\theta), \sin(\theta))$$

$$R_\theta(0,1) = (\cos(\theta + \pi/2), \sin(\theta + \pi/2))$$

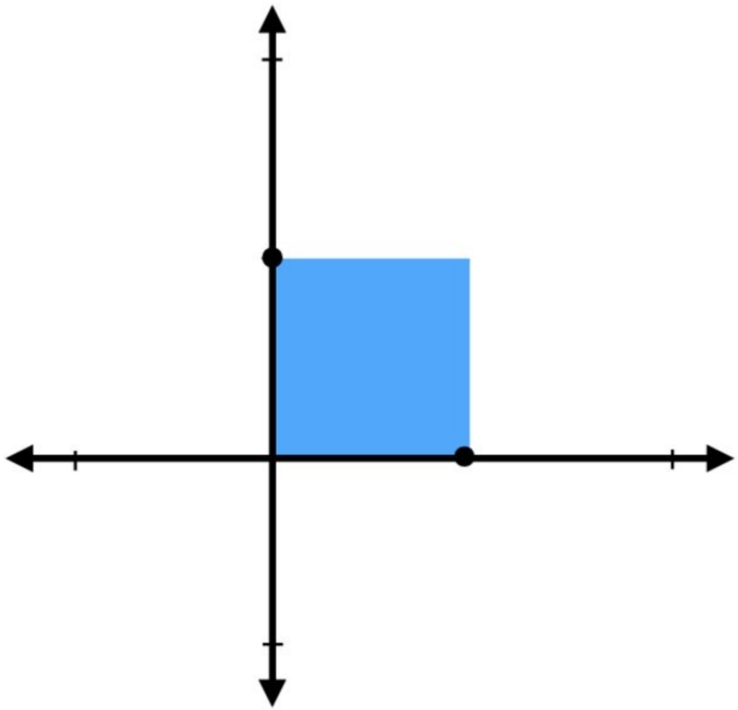
Which means the matrix must look like:

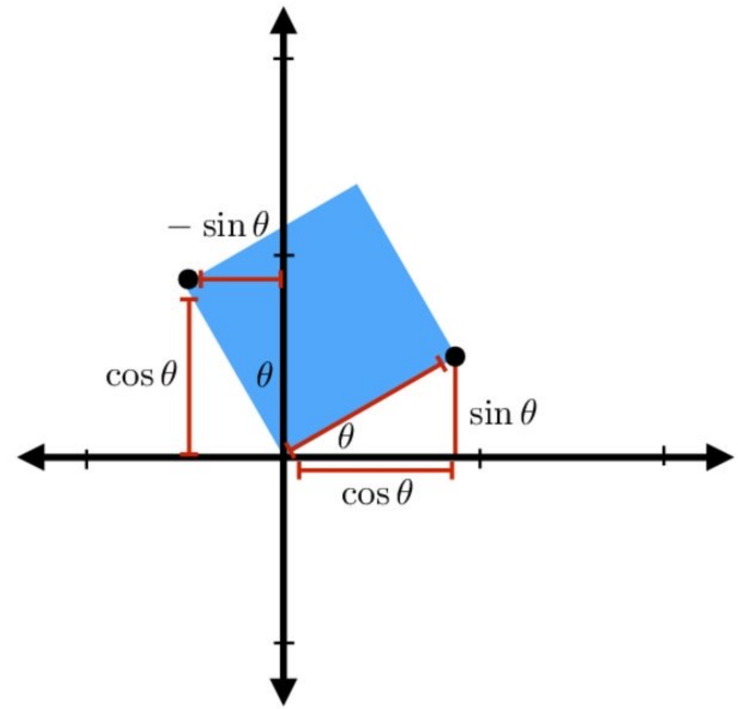
$$R_\theta = \begin{bmatrix} \cos(\theta) & \cos(\theta + \pi/2) \\ \sin(\theta) & \sin(\theta + \pi/2) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

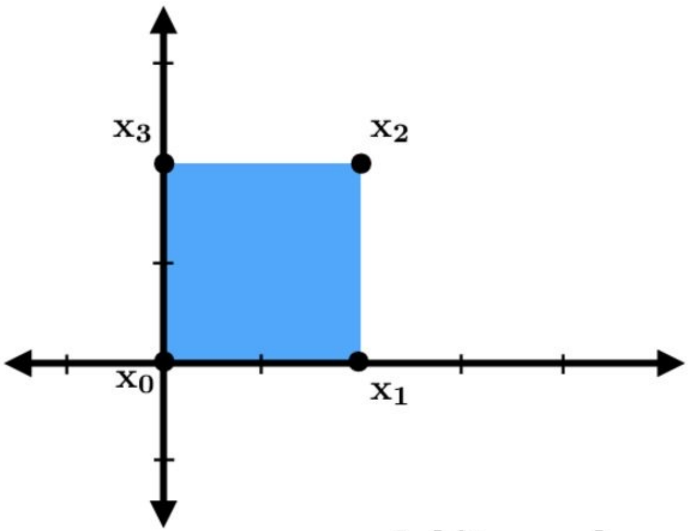
Rotation matrix (2D): another way…

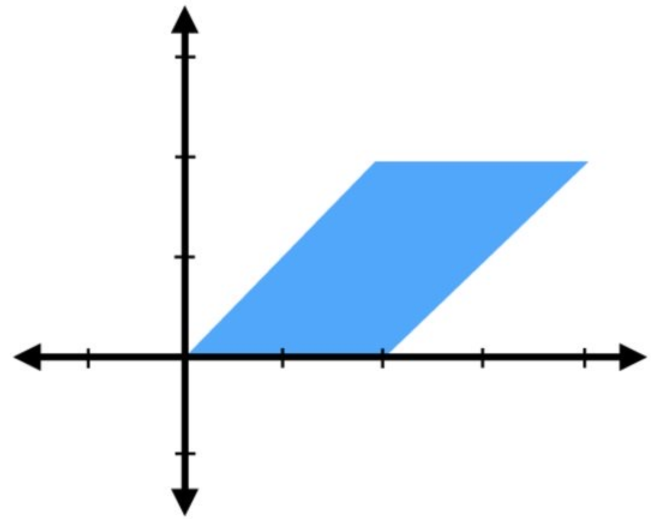
$$\mathbf{R}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$





Shear



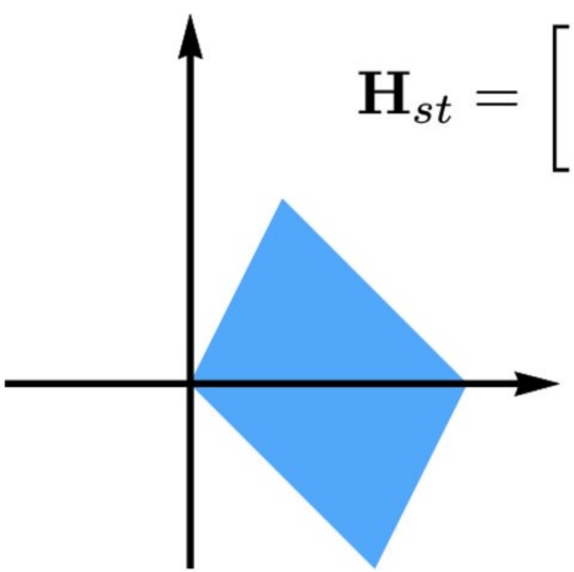


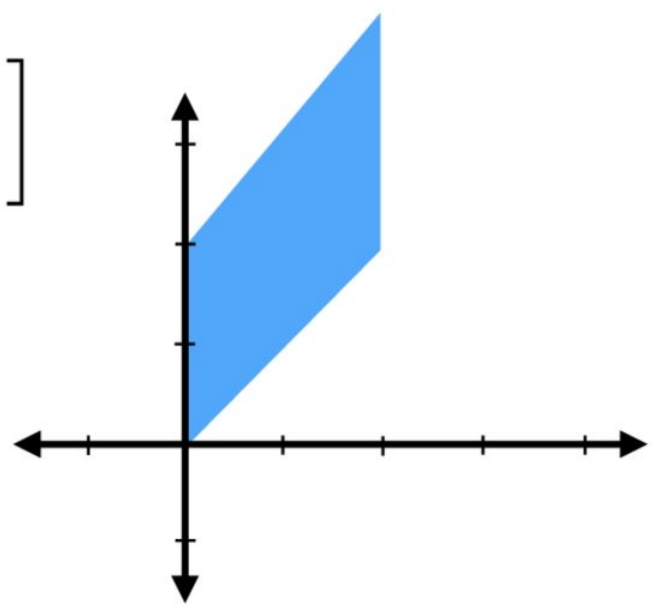
Shear in x:

$$\mathbf{H}_{xs} = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

Arbitrary shear:

$$\mathbf{H}_{st} = \begin{bmatrix} 1 & s \\ t & 1 \end{bmatrix}$$





Shear in y:

$$\mathbf{H}_{ys} = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

Fundamental Transformations

Translation	$\mathbf{p}' = \mathbf{p} + \mathbf{d}$
Scaling	$\mathbf{p}' = \mathbf{S}\mathbf{p}$
Rotation	$\mathbf{p}' = \mathbf{R}\mathbf{p}$

Affine Combinations

- Any affine transformation can be represented as a sequence of these three fundamental transformations
- Because translation isn't a linear transformation, it can't be represented as an $n \times n$ matrix in $\mathbb{R}^n$
- This prevents us from writing affine transformations as concise sequences of $n \times n$ matrix multiplications...

How do we compose transformations?

- Translate then rotate then translate then scale?
- S(R(p+T)+T)
- Not very clean or compressible, homogeneous coordinates are a clever trick to turn this into:
- STRT*p = M*p

matrix

2D *homogeneous* coordinates

2D homogeneous coordinates (2D-H)

Idea: represent 2D points with THREE values ("homogeneous coordinates")

So the point (x, y) is represented as the 3-vector: $\begin{bmatrix} x & y & 1 \end{bmatrix}^T$

And transformations are represented a 3x3 matrices that transform these vectors.

Recover final 2D coordinates by dividing by "extra" (third) coordinate

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow \begin{bmatrix} x/w \\ y/w \end{bmatrix}$$

(More on this later…)

Example: scale and rotation in 2D-H coords

- **For transformations that are already linear, not much changes:**

$$\mathbf{S_s} = \begin{bmatrix} \mathbf{S}_x & 0 & 0 \\ 0 & \mathbf{S}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{R}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Notice that the last row/column doesn't do anything interesting. E.g., for scaling:

$$\mathbf{S_s} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_x x \\ \mathbf{S}_y y \\ 1 \end{bmatrix}$$

Now we divide by the 3rd coordinate to get our final 2D coordinates (not too exciting!)

$$\begin{bmatrix} \mathbf{S}_x x \\ \mathbf{S}_y y \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{S}_x x/1 \\ \mathbf{S}_y y/1 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_x x \\ \mathbf{S}_y y \end{bmatrix}$$

(Will get more interesting when we talk about *perspective*...)

Translation in 2D homogeneous coordinates

Translation expressed as 3x3 matrix multiplication:

$$\mathbf{T_b} = \begin{bmatrix} 1 & 0 & \mathbf{b}_x \\ 0 & 1 & \mathbf{b}_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T_b x} = \begin{bmatrix} 1 & 0 & \mathbf{b}_x \\ 0 & 1 & \mathbf{b}_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_x \\ \mathbf{x}_y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{x}_x + \mathbf{b}_x \\ \mathbf{x}_y + \mathbf{b}_y \\ 1 \end{bmatrix}$$

(remember: just a linear combination of columns!)

Cool: homogeneous coordinates let us encode translations as *linear* transformations!

Transformations in Homogenous Coordinates

Scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} rx \\ sy \\ 1 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ 1 \end{bmatrix}$$

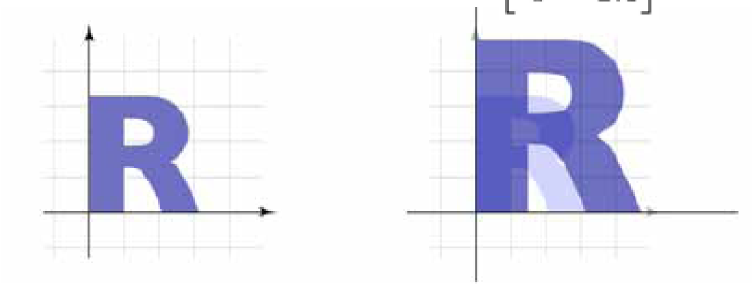
Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+\Delta x \\ y+\Delta y \\ 1 \end{bmatrix}$$

Linear transformation gallery

- Uniform scale $\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} sx \\ sy \end{bmatrix}$

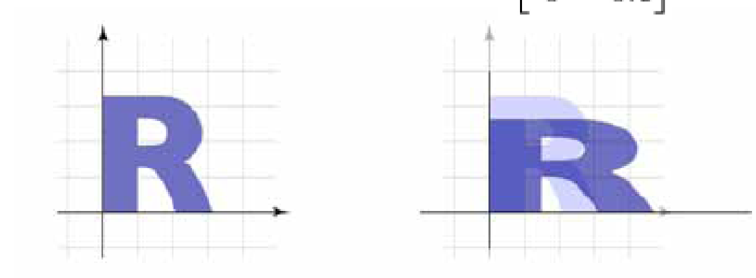
$$\begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$



Linear transformation gallery

- Nonuniform scale $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$

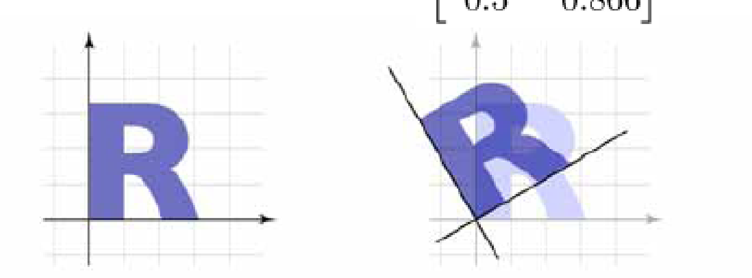
$$\begin{bmatrix} 1.5 & 0 \\ 0 & 0.8 \end{bmatrix}$$



Linear transformation gallery

- Rotation $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$

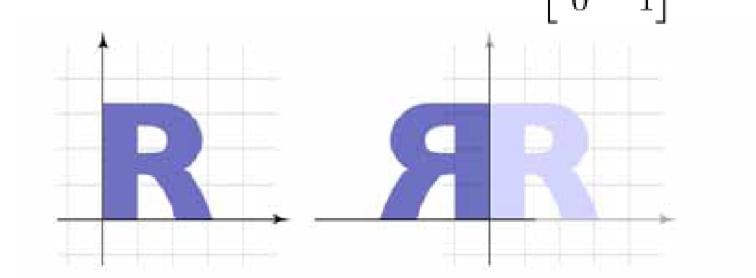
$$\begin{bmatrix} 0.866 & -.05 \\ 0.5 & 0.866 \end{bmatrix}$$



Linear transformation gallery

- Reflection
  - can consider it a special case of nonuniform scale

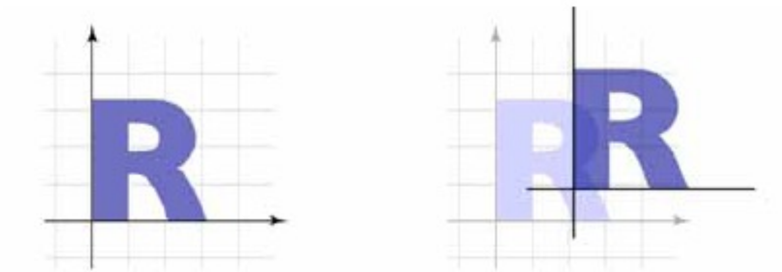
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



Affine transformation gallery

- Translation

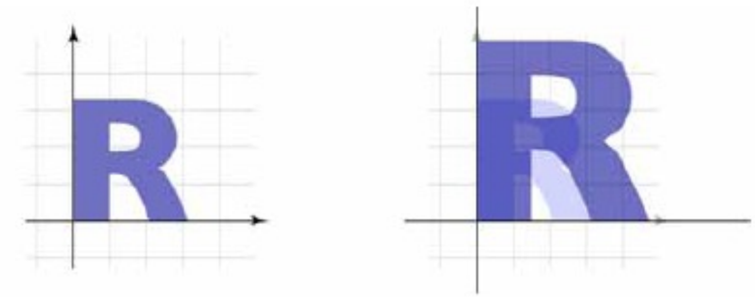
$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2.15 \\ 0 & 1 & 0.85 \\ 0 & 0 & 1 \end{bmatrix}$$



Affine transformation gallery

- Uniform scale

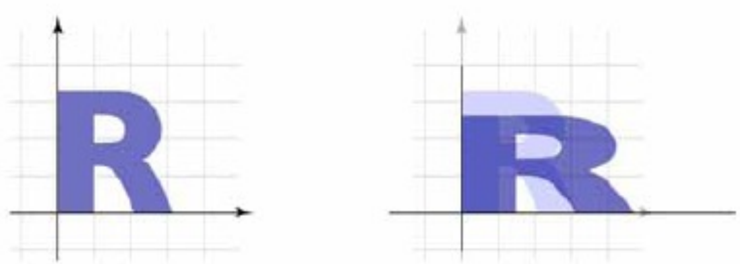
$$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Affine transformation gallery

- Nonuniform scale

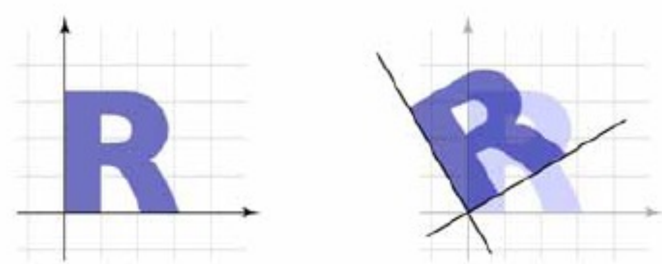
$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



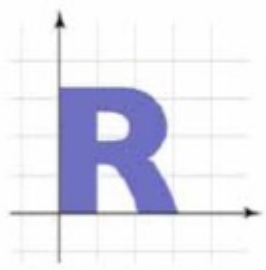
Affine transformation gallery

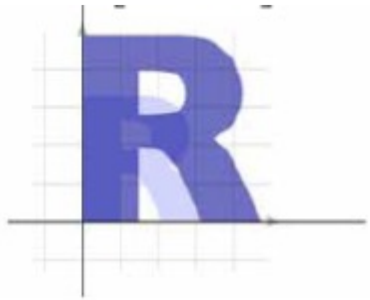
- Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Q on 2D transforms





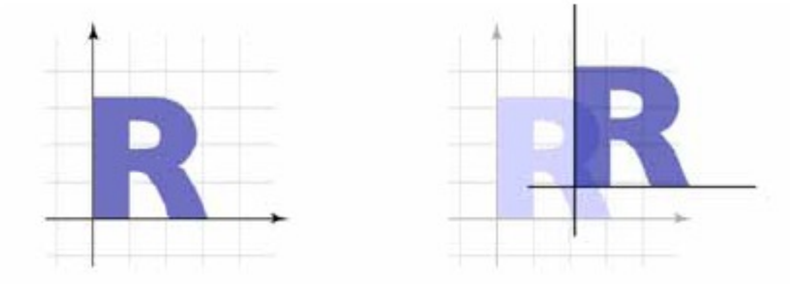
(A) $\begin{bmatrix} 1 & 0 & 2.15 \\ 0 & 1 & 0.85 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q on 2D transforms



(A) $\begin{bmatrix} 1 & 0 & 2.15 \\ 0 & 1 & 0.85 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

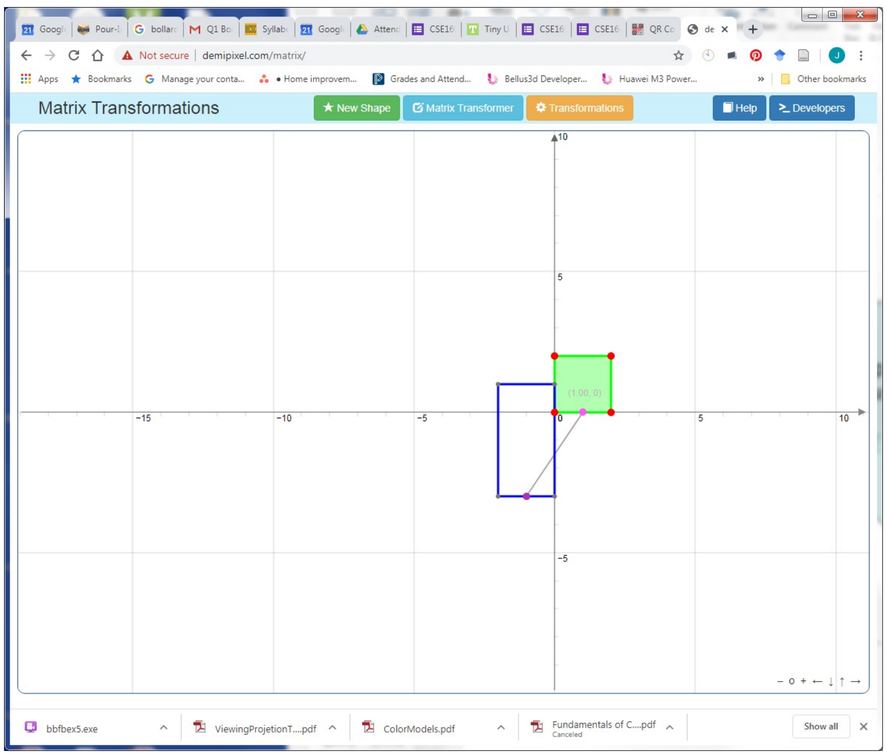
(C) $\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

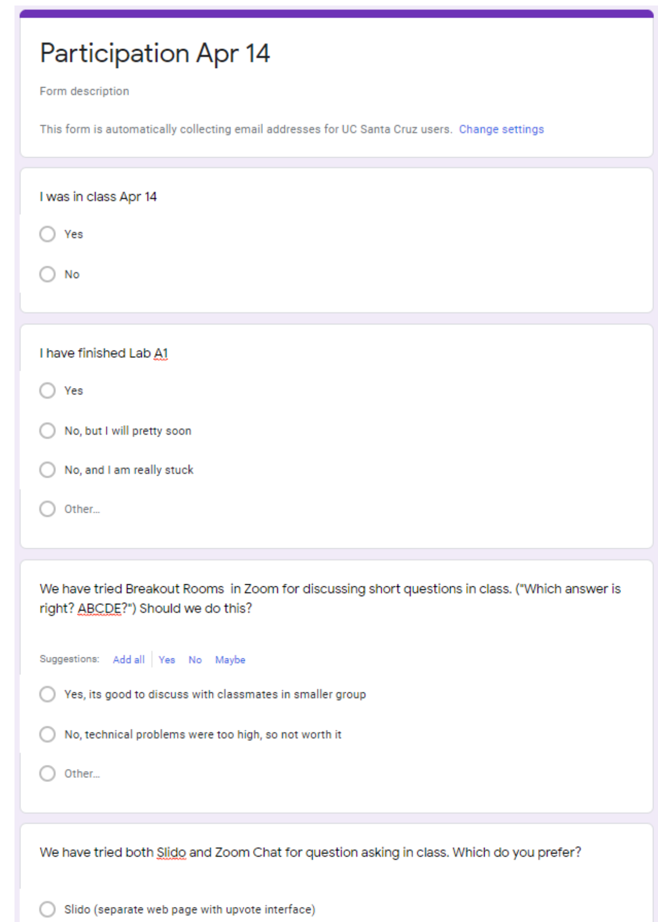
(D) $\begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Participation Survey

Also check out this matrix app
from past student in the class:

http://demipixel.com/matrix/





Participation Apr 14

Form description

This form is automatically collecting email addresses for UC Santa Cruz users. Change settings

I was in class Apr 14

- Yes
- No

I have finished Lab A1

- Yes
- No, but I will pretty soon
- No, and I am really stuck
- Other...

We have tried Breakout Rooms in Zoom for discussing short questions in class. ("Which answer is right? ABCDE?") Should we do this?

Suggestions: Add all | Yes No Maybe

- Yes, its good to discuss with classmates in smaller group
- No, technical problems were too high, so not worth it
- Other...

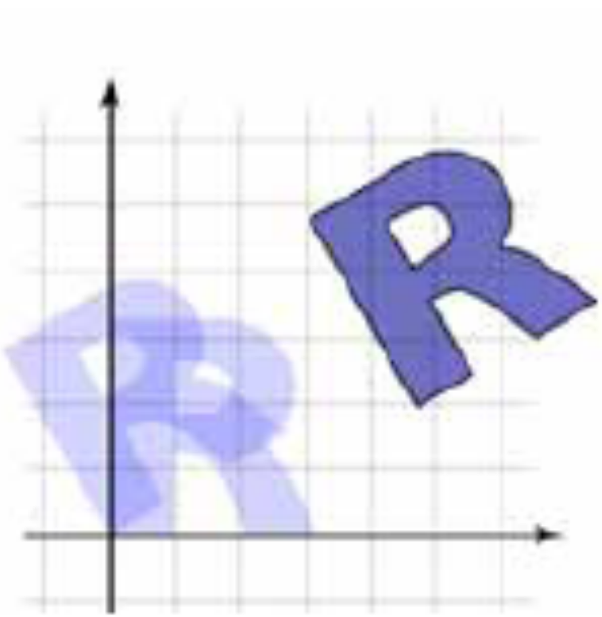
We have tried both Slido and Zoom Chat for question asking in class. Which do you prefer?

- Slido (separate web page with upvote interface)

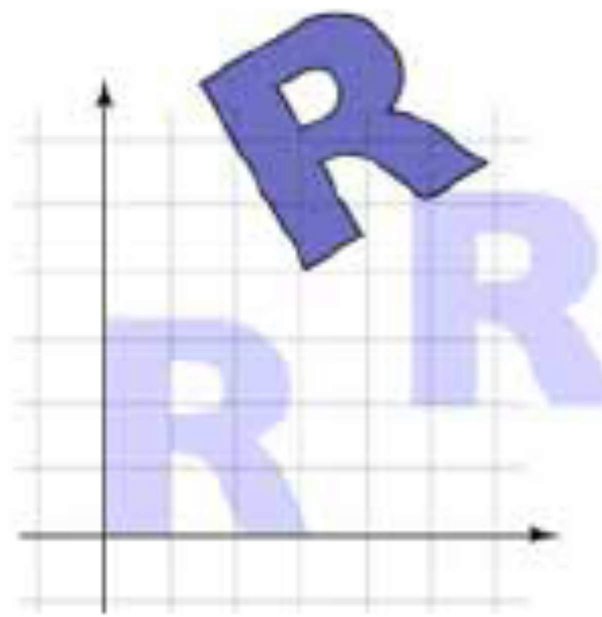
Matrix order matters

Composite affine transformations

- In general **not** commutative: order matters!

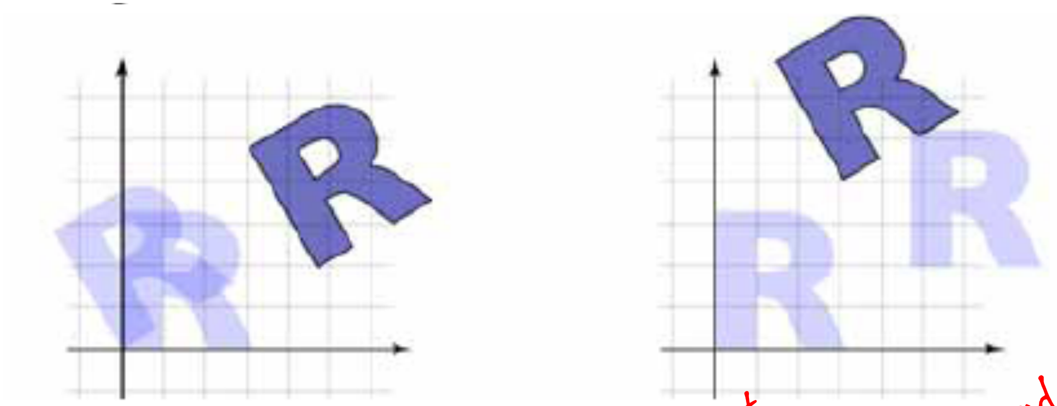


rotate, then translate



translate, then rotate

Multiply matrices in which order?



rotate, then translate

first second
translate, then rotate

p' = [R] [T] p

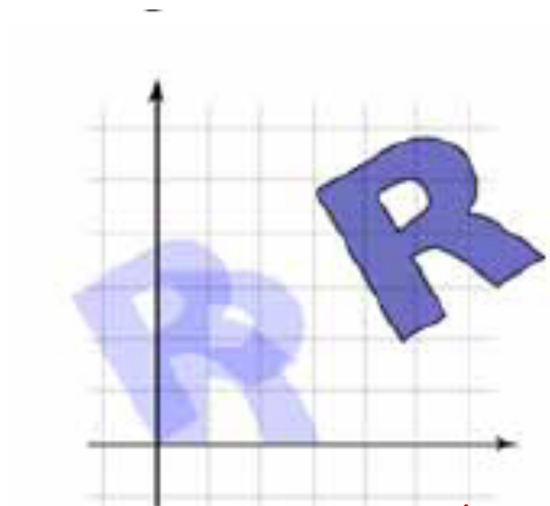
second first

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

OpenGL Right multiplies matrices

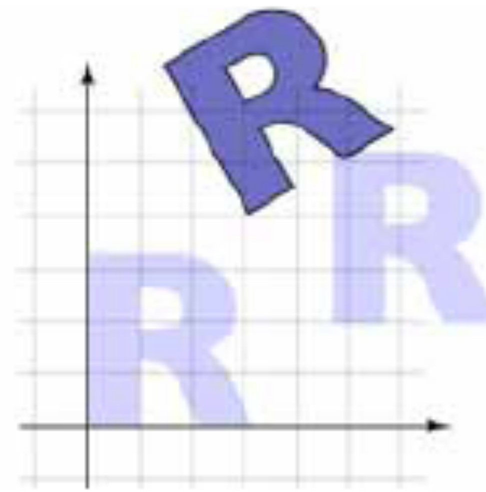


first second

rotate, then translate

p'=[T][R]p

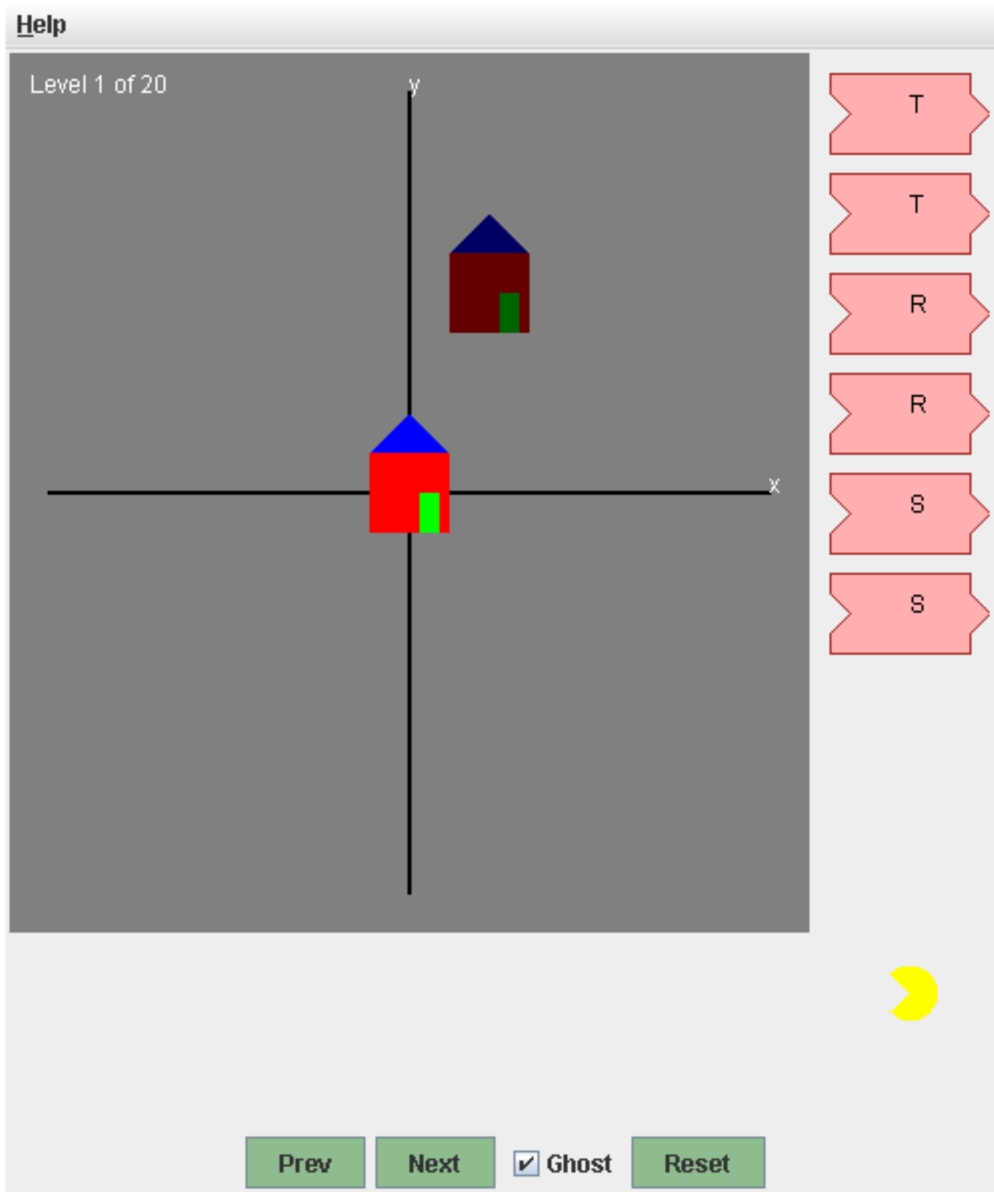
```
Matrix M;
M.translate();   second
M.rotate();      first
draw();
```



translate, then rotate

p'=[R][T]p

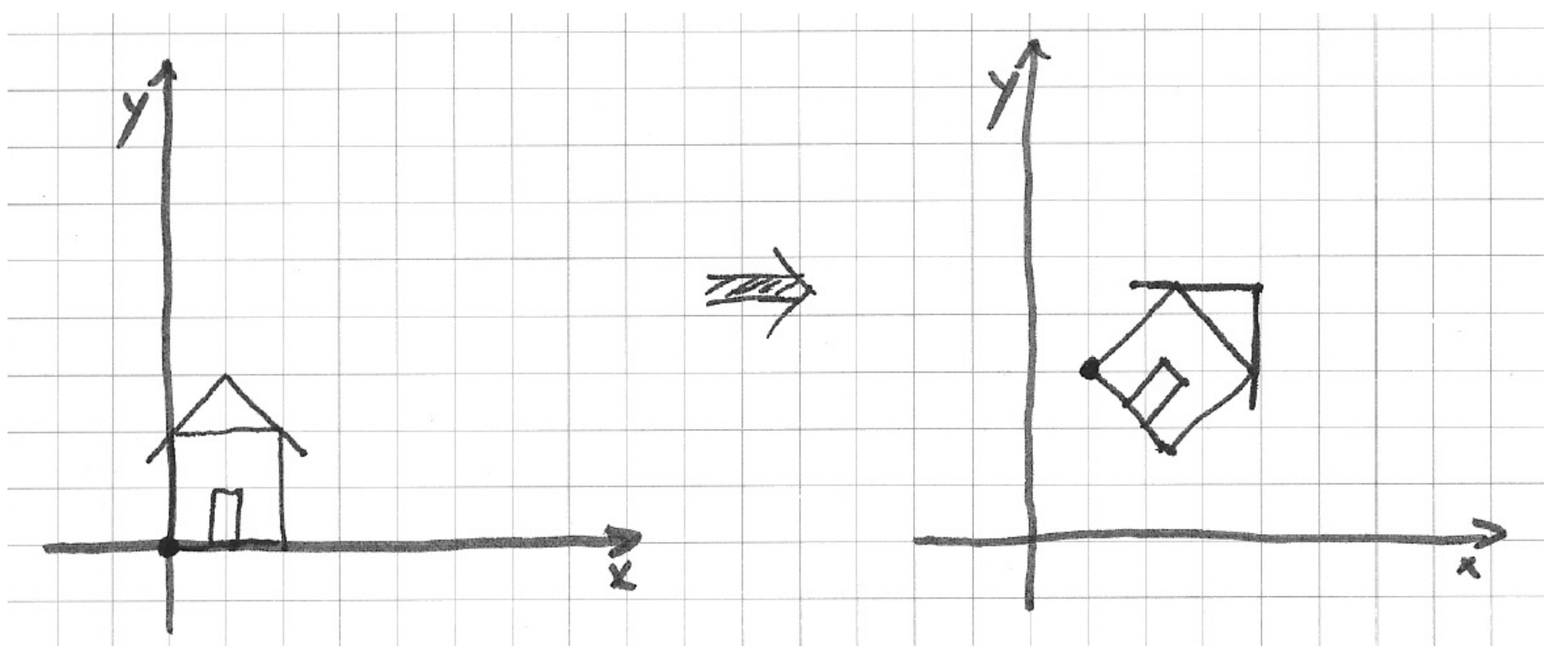
```
Matrix M;
M.rotate();
M.translate();
draw();
```



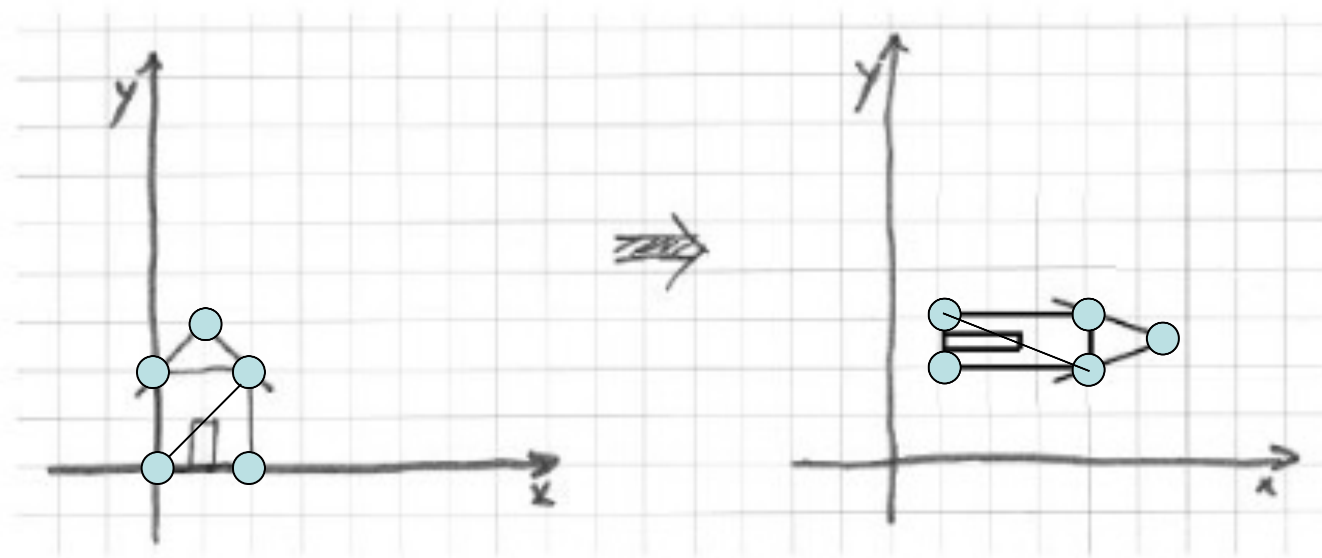
http://www.cs.brown.edu/exploratories/freeSoftware/repository/edu/brown/cs/exploratories/applets/transformationGame/transformation_game_java_browser.html

You need to download the .jar file and run locally

2. What is the matrix necessary to perform the following 2D transformation? (You don't need to actually solve the math, just set up far enough that I would get an actual matrix result if the math was completed.)



Exercise: Find the function()



Vertex buffer array		Vertex buffer array
0,0		1,3
2,2		4,2
2,0	→ f() →	1,2
0,2		4,3
1,3		5.5, 2.5
2,2		4,2
….		….

3D Transformations

Scaling & Translation in 3D

Scaling

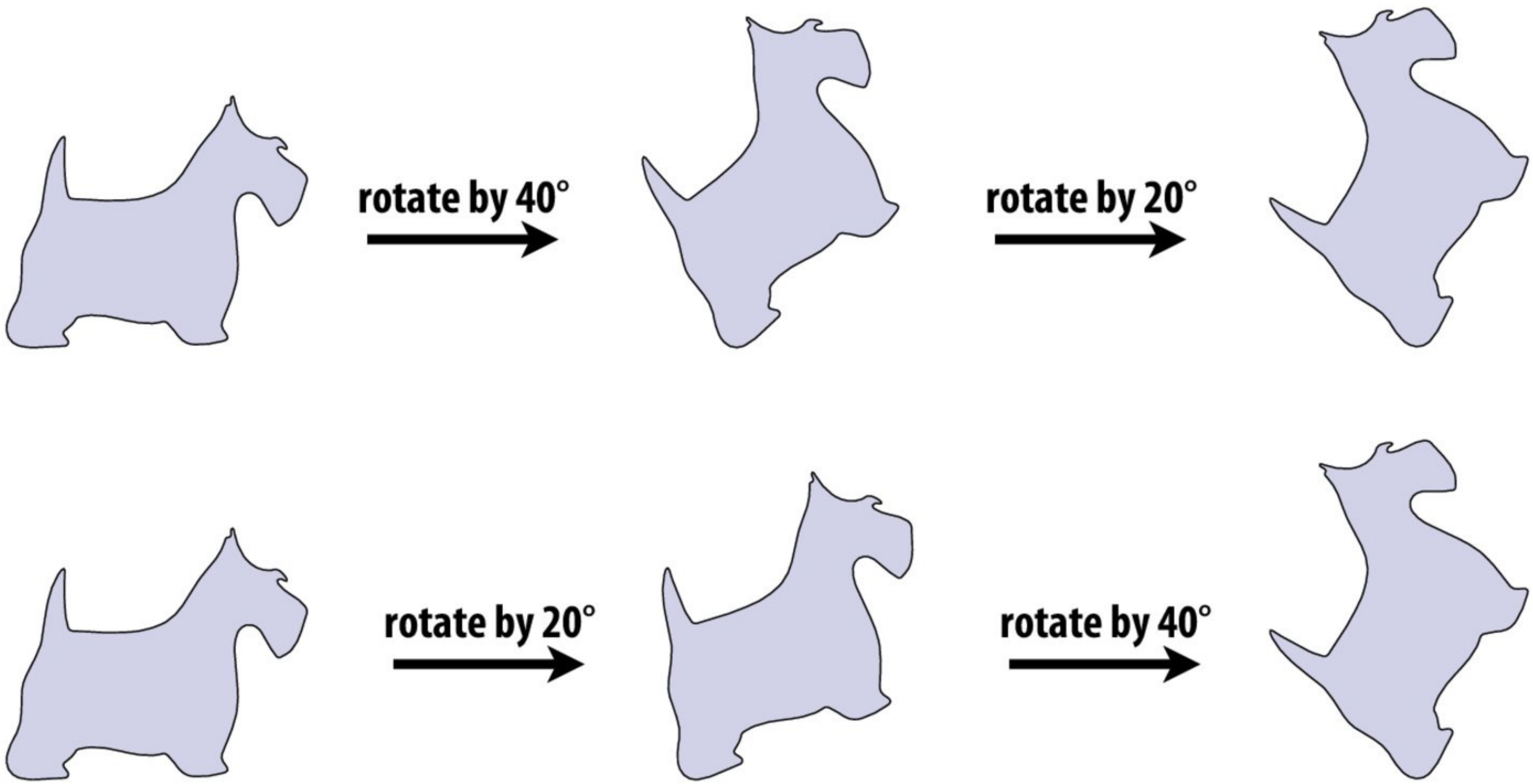
$$\mathbf{S}(r,s,t) = \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation

$$\mathbf{T}(x,y,z) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Commutativity of rotations—2D

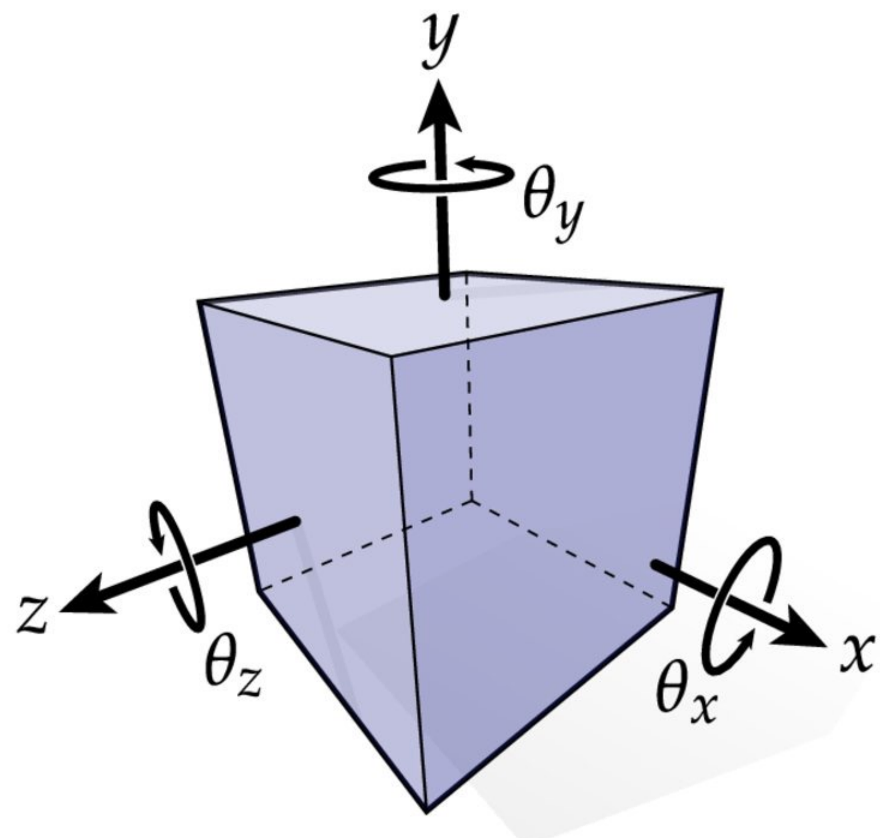
- In 2D, order of rotations doesn't matter:



Same result! ("2D rotations commute")

Representing rotations in 3D—euler angles

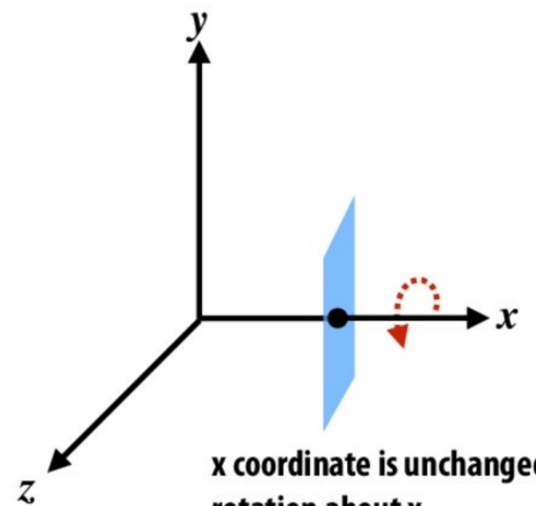
- How do we express rotations in 3D?
- One idea: we know how to do 2D rotations
- Why not simply apply rotations around the three axes? (X,Y,Z)
- Scheme is called *Euler angles*



Rotations in 3D

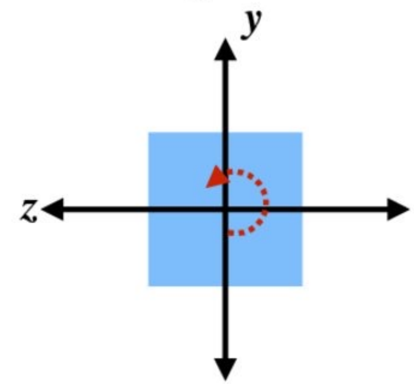
Rotation about x axis:

$$\mathbf{R}_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$



x coordinate is unchanged by rotation about x

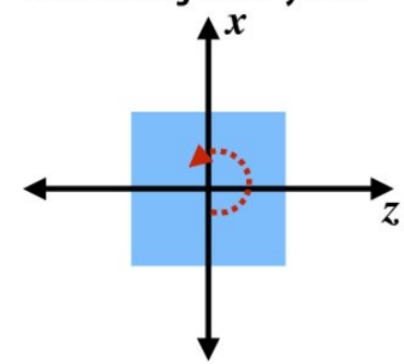
View looking down -x axis:



Rotation about y axis:

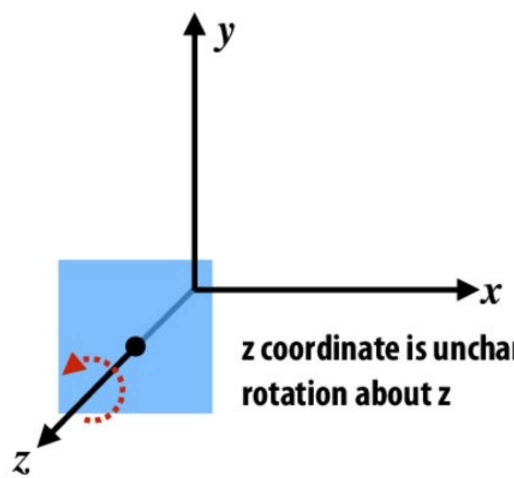
$$\mathbf{R}_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

View looking down -y axis:



Rotation about z axis:

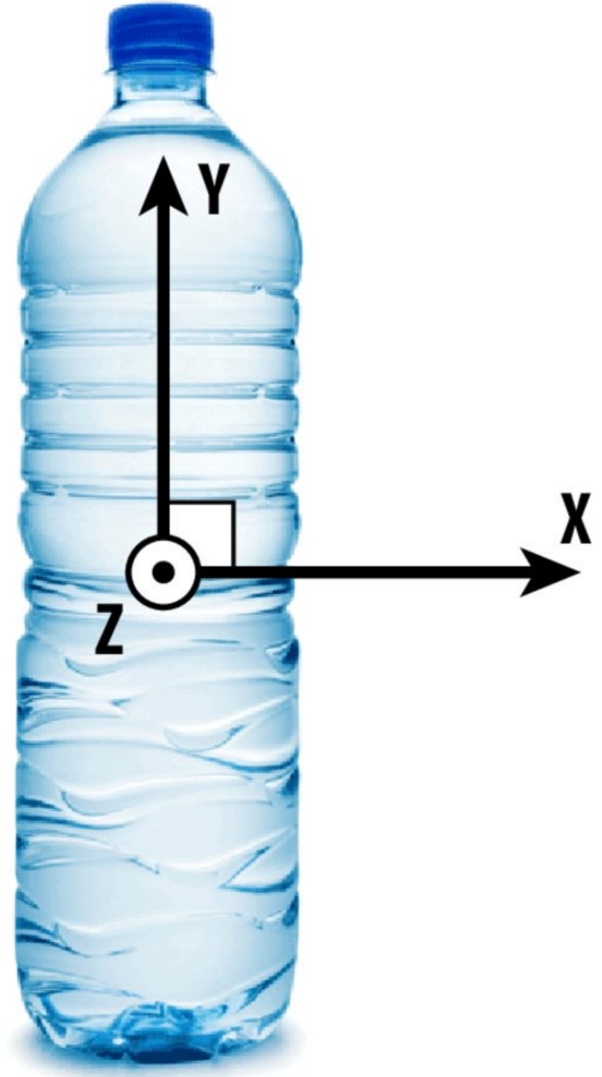
$$\mathbf{R}_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



z coordinate is unchanged by rotation about z

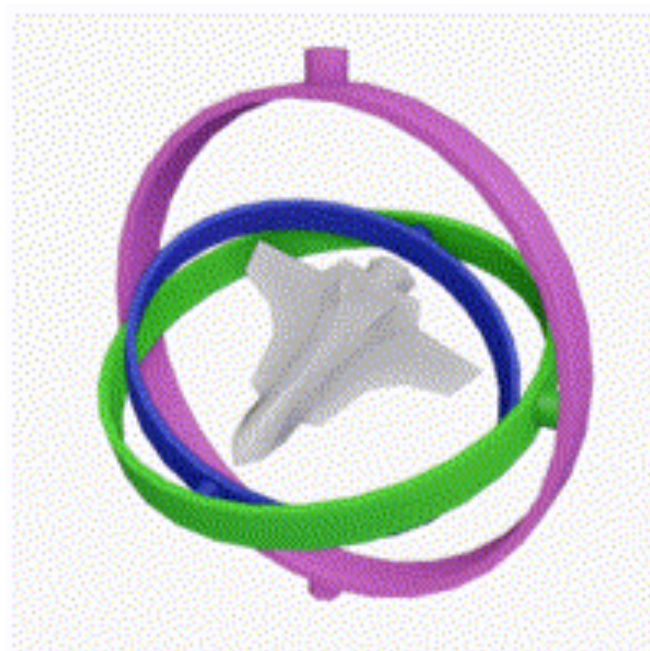
Commutativity of rotations—3D

- What about in 3D?
- IN-CLASS ACTIVITY:
  - Rotate 90° around Y, then 90° around Z, then 90° around X
  - Rotate 90° around Z, then 90° around Y, then 90° around X
  - (Was there any difference?)



CONCLUSION: bad things can happen if we're not careful about the order in which we apply rotations!

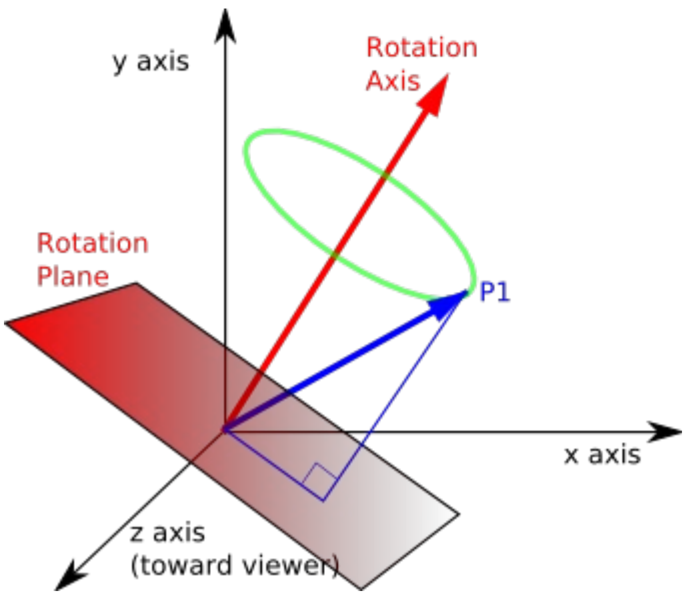
Gimble Lock – lose a degree of freedom



Gimbal locked airplane. When the pitch (green) and yaw (magenta) gimbals become aligned, changes to roll (blue) and yaw apply the same rotation to the airplane. (from Wikipedia)

Alternate rotation : axis-angle

- glRotate(angle, x,y,z);



$\text{gl Rotate}\begin{Bmatrix} f \\ d \end{Bmatrix}(\text{angle}, \underbrace{x, y, z})\,;$

produces this 4×4 matrix

vector around which rotation will occur, +angle ⇔ ccw rotation when looking along vector towards origin.

$$\begin{bmatrix} x^2(1-\cos\theta)+\cos\theta & xy(1-\cos\theta)-z\sin\theta & xz(1-\cos\theta)+y\sin\theta & 0 \\ yx(1-\cos\theta)+z\sin\theta & y^2(1-\cos\theta)+\cos\theta & yz(1-\cos\theta)-x\sin\theta & 0 \\ xz(1-\cos\theta)-y\sin\theta & yz(1-\cos\theta)+x\sin\theta & z^2(1-\cos\theta)+\cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Alternate rotation: Quaternions

Not in this class

Quaternions [edit]

Main article: Quaternions

The complex numbers can be defined by introducing an abstract symbol $\mathbf{i}$ which satisfies the usual rules of algebra and additionally the r
arithmetic: for example:

$$(a+b\mathbf{i})(c+d\mathbf{i}) = ac+ad\mathbf{i}+b\mathbf{i}c+b\mathbf{i}d\mathbf{i} = ac+ad\mathbf{i}+bc\mathbf{i}+bd\mathbf{i}^2 = (ac-bd)+(bc+ad)\mathbf{i}.$$

In the same way the quaternions can be defined by introducing abstract symbols $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ which satisfy the rules $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{i}\mathbf{j}\mathbf{k} = -1$ a
example of such a noncommutative multiplication is matrix multiplication). From this all of the rules of quaternion arithmetic follow, such a
can show that:

$$(a+b\mathbf{i}+c\mathbf{j}+d\mathbf{k})(e+f\mathbf{i}+g\mathbf{j}+h\mathbf{k}) =$$

$$(ae-bf-cg-dh)+(af+be+ch-dg)\mathbf{i}+(ag-bh+ce+df)\mathbf{j}+(ah+bg-cf+de)\mathbf{k}.$$

The imaginary part $b\mathbf{i}+c\mathbf{j}+d\mathbf{k}$ of a quaternion behaves like a vector $\vec{v} = (b,c,d)$ in three dimension vector space, and the real part
convenient to define them as a scalar plus a vector:

$$a+b\mathbf{i}+c\mathbf{j}+d\mathbf{k} = a+\vec{v}.$$

Those who have studied vectors at school might find it strange to add a *number* to a *vector*, as they are objects of very different natures,
if one remembers that it is a mere notation for the real and imaginary parts of a quaternion, it becomes more legitimate. In other words, tl
vector/imaginary part, and another one with zero scalar/real part:

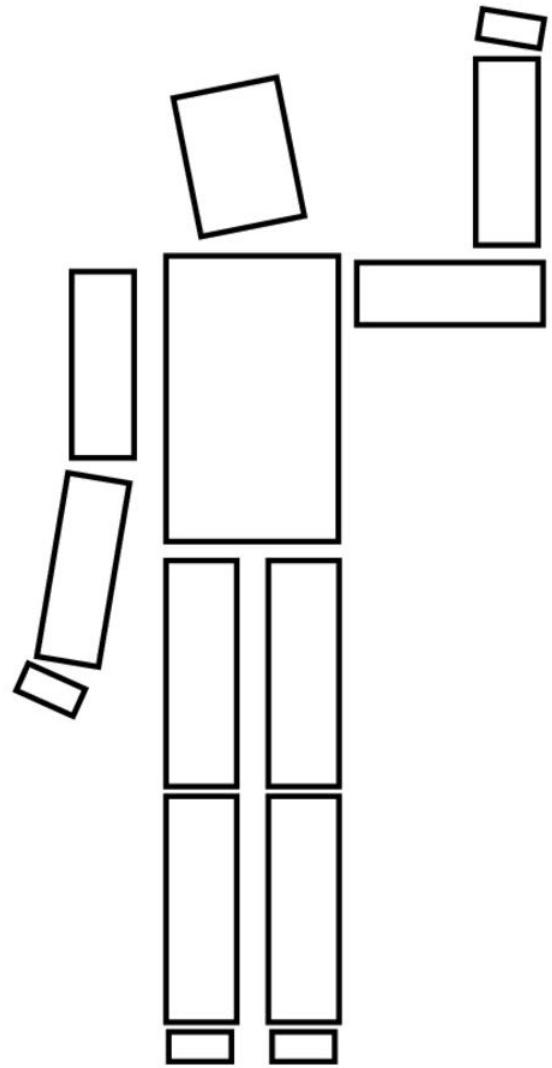
$$a+\vec{v} = (a,\vec{0})+(0,\vec{v}).$$

wikipedia

Heirarchical Transforms

Skeleton - hierarchical representation

- torso
  - head
  - right arm
    - upper arm
      - lower arm
        - hand
  - left arm
    - upper arm
      - lower arm
        - hand
  - right leg
    - upper leg
      - lower leg
        - foot
  - left leg
    - upper leg
      - lower leg
        - foot



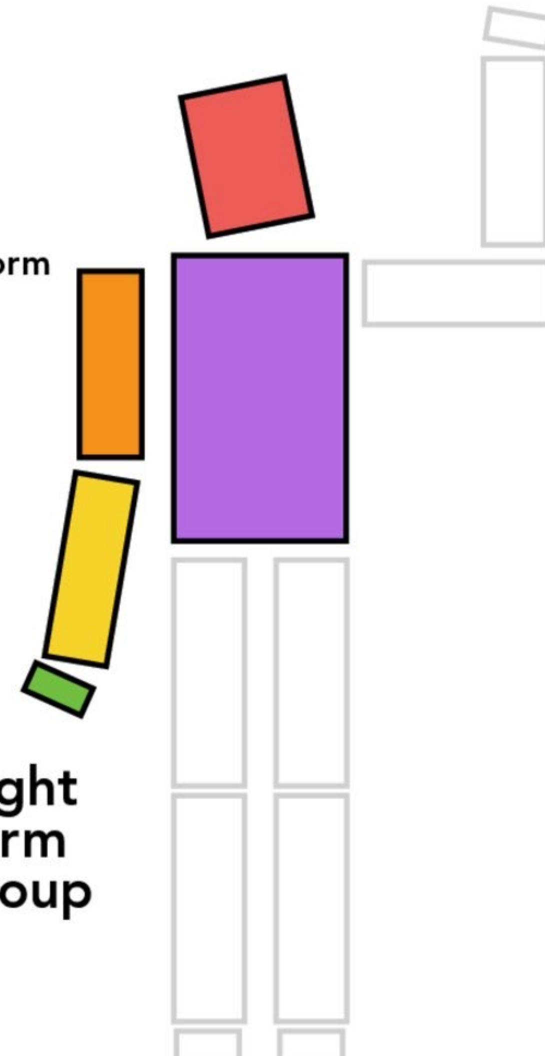
Skeleton - hierarchical representation

```
translate(0, 10);
    drawTorso();
    pushmatrix();  // push a copy of transform onto stack
        translate(0, 5);  // right-multiply onto current transform
        rotate(headRotation);  // right-multiply onto current transform
        drawHead();
    popmatrix();  // pop current transform off stack
    pushmatrix();
        translate(-2, 3);
        rotate(rightShoulderRotation);
        drawUpperArm();
        pushmatrix();
            translate(0, -3);
            rotate(elbowRotation);
            drawLowerArm();
            pushmatrix();
                translate(0, -3);
                rotate(wristRotation);
                drawHand();
            popmatrix();
        popmatrix();
    popmatrix();
    ....
```

right hand

right lower arm group

right arm group



Skeleton - hierarchical representation

```
translate(0, 10);
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    drawHead();
  popmatrix();  // pop current transform off stack
  pushmatrix();
    translate(-2, 3);
    rotate(rightShoulderRotation);
    drawUpperArm();
    pushmatrix();
      translate(0, -3);
      rotate(elbowRotation);
      drawLowerArm();
      pushmatrix();
        translate(0, -3);
        rotate(wristRotation);
        drawHand();
      popmatrix();
    popmatrix();
  popmatrix();
  ....
```
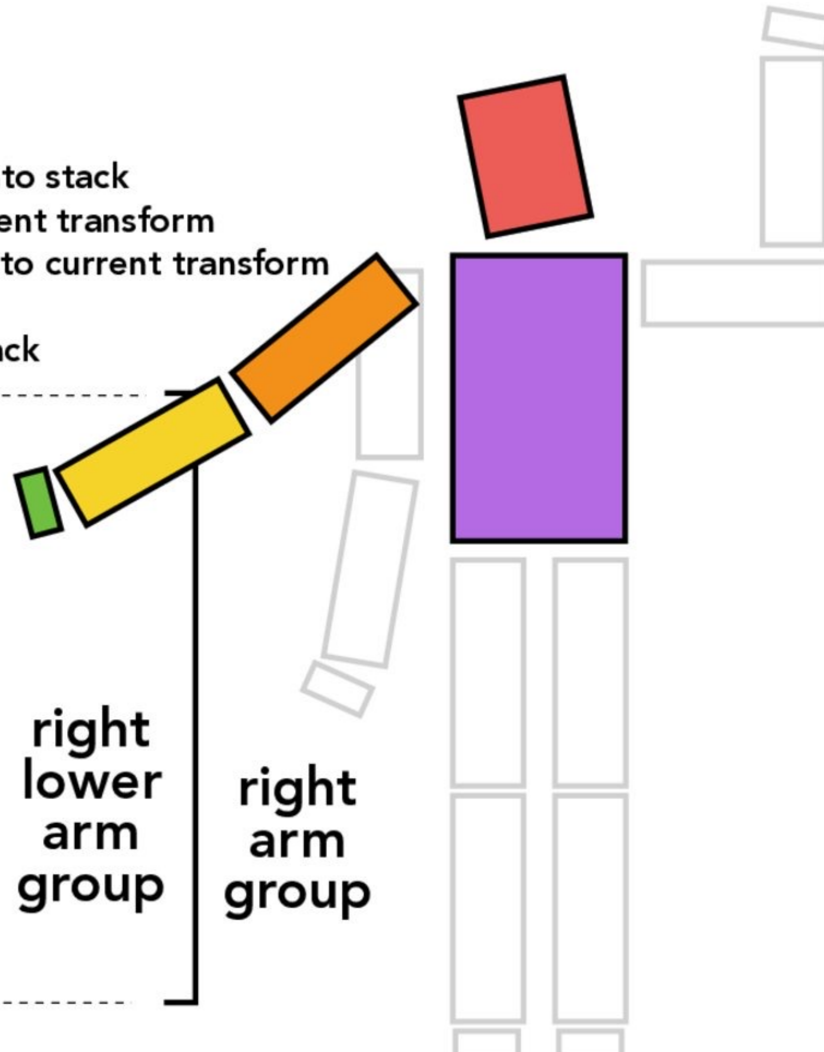

right hand

right lower arm group

right arm group



WebGL transforms

Vertex shader: add a matrix

Listing 3.6 RotatedTriangle_Matrix.js

```
// RotatedTriangle_Matrix.js
// Vertex shader program
var VSHADER_SOURCE =
  'attribute vec4 a_Position;\n' +
  'uniform mat4 u_xformMatrix;\n' +
  'void main() {\n' +
  '  gl_Position = u_xformMatrix * a_Position;\n' +
  '}\n';
```

$P' = [M]P$

Javascript: Pass the matrix in a Uniform

```
function main() {
  ...
  // Set the positions of vertices
  var n = initVertexBuffers(gl);
  ...
  // Create a rotation matrix
  var radian = Math.PI * ANGLE / 180.0; // Convert to radians
  var cosB = Math.cos(radian), sinB = Math.sin(radian);

  // Note: WebGL is column major order
  var xformMatrix = new Float32Array([
     cosB, sinB, 0.0, 0.0,
    -sinB, cosB, 0.0, 0.0,
     0.0,  0.0, 1.0, 0.0,
     0.0,  0.0, 0.0, 1.0
  ]);

  // Pass the rotation matrix to the vertex shader
  var u_xformMatrix = gl.getUniformLocation(gl.program, 'u_xformMatrix');
  ...
  gl.uniformMatrix4fv(u_xformMatrix, false, xformMatrix);

  // Set the color for clearing <canvas>
  ...
  // Draw a triangle
  gl.drawArrays(gl.TRIANGLES, 0, n);
}
```

Rotation matrix

Send to GPU

Draw

Listing 4.2 RotatedTranslatedTriangle.js

```
// RotatedTranslatedTriangle.js
// Vertex shader program
var VSHADER_SOURCE =
  'attribute vec4 a_Position;\n' +
  'uniform mat4 u_ModelMatrix;\n' +
  'void main() {\n' +
  '  gl_Position = u_ModelMatrix * a_Position;\n' +
  '}\n';
// Fragment shader program
...
function main() {
  ...
  // Set the positions of vertices
  var n = initVertexBuffers(gl);
  ...
  // Create Matrix4 object for model transformation
  var modelMatrix = new Matrix4();

  // Calculate a model matrix
  var ANGLE = 60.0; // Rotation angle
  var Tx = 0.5; // Translation distance
  modelMatrix.setRotate(ANGLE, 0, 0, 1); // Set rotation matrix
  modelMatrix.translate(Tx, 0, 0); // Multiply modelMatrix by the calculated
                                   ➥translation matrix

  // Pass the model matrix to the vertex shader
  var u_ModelMatrix = gl.getUniformLocation(gl.program, 'u_ModelMatrix');
  ...
  gl.uniformMatrix4fv(u_ModelMatrix, false, modelMatrix.elements);
  ...
  // Draw a triangle
  gl.drawArrays(gl.TRIANGLES, 0, n);
}
```

Composing multiple matrices, but still just one in vertex shader

[M] = identityMatrix

[M] = [R]

[M] = [R][T]

Send to GPU

Draw

Administrative

Q&A

End